- Interesting case (later): $k=1$. Here we need in addition the analysis of $Mp(2,\mathbb{Z})$-module $M_{1/2}$.

<u>Questions</u>

a) Which thetablocks are Jacobi forms (i.e. hol. in $\mathfrak{H}\times\mathbb{C}$ and at ∞)?

b) Which Jacobi forms are thetablocks?

c) "How many" Jacobi forms does one obtain by thetablocks?

For a) we study

TB = group of thetablocks

Holomorphic ones form semi-subgroup. Want to characterize these.

divisor of $\vartheta_d(\tau,\cdot)$ (fixed τ, as theta function on $\mathbb{C}/\mathbb{Z}\tau+\mathbb{Z}$)

= d-division pts of $\mathbb{C}/\mathbb{Z}\tau+\mathbb{Z}$

divisor of $f(\tau,\cdot) = \sum_a n_a \Pi_a$, Π_a = formal sum of primitive a-division pts of $\mathbb{C}/\mathbb{Z}\tau+\mathbb{Z}$

Define

$$\mathrm{Div}(f) := \sum_a n_a (a) \in \mathbb{Z}[\mathbb{Z}_{>0}]$$

$f \mapsto \mathrm{Div}(f)$ defines exact sequence

<u>Prop</u> 1) $1 \to \eta^{\mathbb{Z}} \to TB \xrightarrow{\mathrm{Div}} \mathbb{Z}[\mathbb{Z}_{>0}] \to 1$

2) The sequence splits via

$$\sum n_a (a) \longmapsto \prod_a \Big(\underbrace{\sum_{d\mid a} \vartheta_d^{\mu(a/d)}}_{S_a}\Big)^{n_a}$$

<u>Note</u> f holomorphic in $\mathfrak{H}\times\mathbb{C}$ iff $\mathrm{Div}(f)\geq 0$

(equiv. f = product of S_a's)