~~[illegible]~~ : $\vartheta_d := \vartheta(\tau, dz)$ $\qquad (d \geq 1)$

Thetablocks (B) $f := \eta^{?}\, \dfrac{\vartheta_{a_1}^{u_1} \cdots \vartheta_{a_r}^{u_r}}{\vartheta_{b_1} \cdots \vartheta_{b_s}}$ $\quad$ ( ~~[illegible]~~ $u \in \mathbb{Z}$, $a_i, b_j \in \mathbb{Z}_{\geq 1}$ )

Thm $J_{\frac12, \frac n2}(\varepsilon^a \ldots) = 0$ $\quad$ for all $n, a, b$ except for

$$J_{\frac12, \frac{d^2}{2}}(\varepsilon^3) = \mathbb{C}\, \vartheta_d \qquad (d \geq 1)$$

$$J_{\frac12, \frac{3d^2}{2}}(\varepsilon^d) = \mathbb{C}\, \frac{\vartheta_{2d}}{\vartheta_d} \cdot \eta \qquad (d \geq 1).$$

Note: $\dfrac{\vartheta_2}{\vartheta}\,\eta = \sum_{r \in \mathbb{Z}} \left(\dfrac{12}{r}\right) q^{r^2/24} \zeta^{r/2}$ $\quad$ (Nullwert = Dedekind $\eta$)

$$= \eta(\tau) + O(z^2)$$

proof: theta-expansion

$\phi \in J_{k,m}(\varepsilon^a)$ : $\qquad \phi(\tau, z) = \sum h_\rho(\tau)\, \vartheta_{m,\rho}(\tau, z)$

say $m \in \mathbb{Z}$ (otherwise consider $\phi(\tau, 2z) \in J_{k,4m}(\varepsilon^a)$ )

$\vartheta_{m,\rho} = \sum_{r \equiv \rho\,(2m)} q^{r^2/4m} y^r \qquad \rho \bmod 2m$

$Th_m := \operatorname{span}\langle \vartheta_{m,\rho} \mid \rho \bmod 2m \rangle$ is $Mp(2,\mathbb{Z})$-module (w.r.t. $|_{\frac12, m}$ )

$h_\rho \in M_{k-\frac12} := \sum_{n \geq 1} M_{k-\frac12}(\Gamma(4n))$.

Conclusion:

$$J_{k,m}(\varepsilon^a) \cong \left(M_{k-\frac12} \otimes Th_m\right)^{Mp(2,\mathbb{Z}),\, \varepsilon^a}$$

In partic.

$$J_{\frac12, m}(\varepsilon^a) = Th_m^{\,Mp(2,\mathbb{Z}),\, \varepsilon^a}$$

Thm (S 1985) $Th_m$ contains a 1-dim. $Mp(2,\mathbb{Z})$-submodule only if $m = 2d^2$ (and then exactly one, which equals $\vartheta_{2d}$ ) or $m = 6d^2$ ( $\underline{\qquad}$ $\dfrac{\vartheta_{4d}}{\phantom{x}}\,\eta$ ). □