

定理 5.1 次が成立する.

(1)(Tsuyumine [46]) $n = 2, d > 1$ のとき, $M(\Gamma_K)^{(d)}$ は Cohen-Macaulay.

(2)(Freitag [52]) $n \geq 3, d \geq 1$ のとき, $M_{\text{ev}}(\Gamma_K)^{(d)}$ は Cohen-Macaulay でない.

(2) よりも弱い結果として, $n = 3$ のとき, $M_{\text{ev}}(\Gamma_K)$ は Gorenstein でない (Thomas-Vasquez [45]), n が 3 以上の奇数のとき, $M_{\text{ev}}(\Gamma_K)$ は Gorenstein でない (Feng [7]).

次元公式: $S_k(\Gamma_K)$ については Shimizu [44], Freitag [9] を, $S_k^s(\Gamma_K)$ については Busam [2] を参照されたい.

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