定理 4.11 (van der Geer [11]) $H \times H_-$ 上で考えたとき, weight 1, 2, 3, 4, 6 の保型形式からなる生成系がある：

$$M(\Gamma_K) = \mathbb{C}\left[\frac{\sigma_3}{\sqrt{\sigma_2}}, \sigma_2, \frac{\delta}{\sqrt{\sigma_4}}, \sigma_4, \Delta\right].$$

ただし,

- $\Delta^2 = 256\sigma_4^3 - 128\sigma_4^2\sigma_2^2 + 16\sigma_4\sigma_2^4 + 144\sigma_4\sigma_3^2\sigma_2 - 27\sigma_3^4 - 4\sigma_3^2\sigma_2^3$,
- $\delta^2 = \sigma_2(9\sigma_3^2\sigma_2 - \sigma_4\sigma_2^2 + 12\sigma_4^2)$.

$$\sum_{k=0}^{\infty} \dim M_k(\Gamma_K)t^k = \frac{1 - t + t^3 - t^5 + t^6}{(1-t)^2(1-t^2)}.$$

4.5 $K = \mathbb{Q}(\sqrt{13})$ の場合

$0 = [A_2, B_4, B_4^3]$
$= [A_2, B_4, C_6C_6']$
$= [A_2, B_4, C_6] + C_6'[A_2, B_4, C_6]$
$= C_6[A_2, B_4, C_6']$

4.5.1 Symmetric, even weight case

定理 4.12 (van der Geer-Zagier [12])

$\therefore \frac{[A_2, B_4, C_6]}{C_6} = -\frac{[A_2, B_4, C_6']}{C_6'}$

$$M_{\text{ev}}^s(\Gamma_K) = \mathbb{C}[A_2, B_4, C_6, C_6'].$$

ただし, $B_4^3 = C_6C_6'$.

$$\sum_{k=0}^{\infty} \dim M_{2k}^s(\Gamma_K)t^{2k} = \frac{1 + t^4 + t^8}{(1-t^2)(1-t^6)^2}.$$

$2+4+6+2=14$
$14-6=8$
$D_8 = [A_2, B_4, C_6]/C_6$?

4.5.2 Even weight case

定理 4.13 (van der Geer-Zagier [12])

$$M_{\text{ev}}(\Gamma_K) = \mathbb{C}[A_2, B_4, C_6, C_6', D_8].$$

ただし,

$$\begin{aligned}
B_4^3 &= C_6C_6', \\
D_8^2 &= 256A_2^5C_6 - 128A_2^4B_4^2 + 16A_2^3B_4D_8 \\
&\quad -656A_2^3B_4C_6 + 776A_2^2B_4^3 - 261A_2B_4^2D_8 \\
&\quad +27B_4D_8^2 - 27A_2^2C_6^2 + \frac{495}{2}A_2B_4^2C_6 - \frac{947}{16}B_4^4 \\
&\quad +54B_4C_6^3,
\end{aligned}$$

$$\sum_{k=0}^{\infty} \dim M_{2k}(\Gamma_K)t^{2k} = \frac{1 - t^2 + t^4 + t^8 - t^{10} + t^{12}}{(1-t^2)^2(1-t^6)}. = \frac{(1+t^4+t^8)(1+t^8)}{(1-t^2)(1-t^6)^2}$$