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On the Graded Rings of Modular Forms in Several Variables

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The theory of modular forms in one variable has a long history. Gauß already knew modular functions in one variable in connection with elliptic functions (Takagi [49]). The theory of modular forms in several variables is younger than it, but is now nearly a century old. It was noted by Hilbert at the end of the last century, and first treated by his student Blumenthal [2]. In 1935 Siegel introduced a new class of modular forms in several variables in a celebrated paper [47]. Nowadays the theory of automorphic forms on a bounded symmetric domain has been developed in several directions as well as the theory of automorphic forms in a rather wider class. However, in spite of this long history, there is still only a small number of examples of rings of automorphic forms in several variables for which much is known about the structure. Here we give a list of papers concerning the structure of the graded ring, the generators or the relations among them, which is arranged nearly chronologically in each case:

Siegel modular forms; Igusa [28, 29, 30], Hammond [15], Freitag [6], Tsuyumine [56], Satoh [41].

Hilbert modular forms; Gundlach [13, 14], Hammond [16], Fomenko [5], Resnikoff [36], Hirzebruch [21, 22, 23], Van der Geer [11], Van der Geer-Zagier [12], Hermann [17, 18], Nagaoka [35], Müller [32, 33].

Hermitian modular forms; Freitag [7].

Automorphic forms on the complex 2-ball; Resnikoff-Tai [37, 48].

There are some other papers concerning the structure of a ring of modular forms over Z. Here we should mention that the dimension formulas for spaces of modular forms, or rather cusp forms, are known for wider class. They have helped, and will keep to help, the work of determining the structure of a graded ring. Shimizu's formula [45] has been continually employed to study a graded ring of Hilbert modular forms (mainly in two dimensional case yet), and the formula by Busam [3] in the symmetric case. The works by Resnikoff-Tai [37, 48] were done by

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