Proof. From Lemma 4 and Lemma 9 (3), The dimension of the $A_{k,1}$ is not greater than the coefficient of x^k on the formal power series development.

$$\frac{x^6}{(1-x^2)(1-x^3)(1-x^5)} - \sum_{n=0}^{\infty} x^{5n+6} = \frac{x^8+x^9-x^{11}}{(1-x^2)(1-x^3)(1-x^5)}$$
$$= \frac{x^8+x^9+x^{12}-x^{14}}{(1-x^2)(1-x^5)(1-x^6)}.$$

□

4 Remark

Our estimation is quite rough. Generally, this estimation is not good for large l and d_K. In fact, this article and the exposition [1] show the following results.
(1) If $d_K = 5$, this estimation is sharp if $l = 0, 2$. When $l = 1$, this estimation is 'quite' sharp: we know the true dimension from this estimation easily.
(2) If $d_K = 12$, this estimation is sharp only if $l = 0$. When $l = 1$, this estimation is 'quite' sharp.
(3) If $d_K = 8$ this estimation is sharp only if $l = 0$. Even if $l = 1$, this estimation is not sharp.
In other cases, to determine the true dimension from this estimation, the author think that we need more ideas.

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References

[1] H. Aoki, *Estimate of the dimension of Hilbert modular forms by means of differential operator*, Automorphic forms and Zeta functions (Rikkyo, 2004), Proceedings of the Conference in Memory of Tsuneo Arakawa, World Scientific, 20-28.

[2] H. Aoki and T. Ibukiyama, *Simple Graded Rings of Siegel Modular Forms, Differential Operators and Borcherds Products*, Int. J. Math., **16-3**, 2005, 249–279.

[3] M. Eichler and D. Zagier, *The Theory of Jacobi Forms*, Progress in Math. **55**, Birkhäuser, Boston, 1985.

[4] E. Freitag, Hilbert modular forms, Springer, 1990.

[5] K. B. Gundlach, *Die Bestimmung der Funktionen zur Hilbertschen Modulgruppe des Zahlkörpers* $\mathbb{Q}(\sqrt{5})$, Math. Ann., **152**, 1963, 226–256.