because $\frac{3}{2}n + \frac{1}{2}v < n$. Hence $F \in A_{k,l}(n)$ means

$$\sum_{v=0}^{n} v^r c(n, -n+2r) = 0 \quad (r = 0, 1, \dots, n-1).$$

Easily we have $c(n,n) = (-1)^{k+l}\varepsilon^{2l} c(n,-n)$, hence

$$\begin{pmatrix} -1 & 0 & \dots & 0 & (-1)^{k+l}\varepsilon^{2l} \\ 1 & 1 & \dots & 1 & 1 \\ n & n-2 & \dots & -n+2 & -n \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ n^{n-2} & (n-2)^{n-2} & \dots & (-n+2)^{n-2} & (-n)^{n-2} \\ n^{n-1} & (n-2)^{n-1} & \dots & (-n+2)^{n-1} & (-n)^{n-1} \end{pmatrix} \begin{pmatrix} c(n,n) \\ c(n,n-2) \\ c(n,n-4) \\ \vdots \\ c(n,-n+2) \\ c(n,-n) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}.$$

By the Vandermonde formula, we have $c(n,v) = 0$. □

Proposition 5. *The dimension of the $A_{k,l}$ is not greater than the coefficient of x^k on the formal power series development.*

$$\frac{x^6}{(1-x^2)(1-x^3)(1-x^5)}$$

Proof. From the previous lemma, the dimension of $A_{k,l}$ is not greater than

$$\sum_{n=0}^{\infty} \dim M_{2k+2n}(n+1) = \sum_{n=0}^{\infty} \dim M_{2(k-5n-6)}.$$

This is the coefficient of x^k on the formal power series development of

$$\sum_{n=0}^{\infty} \frac{x^{5n+6}}{(1-x^2)(1-x^3)} = \frac{x^6}{(1-x^2)(1-x^3)(1-x^5)}.$$

□

3 Construction

M.H.Lee [12] gave the way to construct mixed weight Hilbert modular forms by using generalized Rankin-Cohen operator. Especially, for $F_j \in A_{k_j,0}$, we have

$$[F_1, F_2]_1 := \frac{1}{2\pi\sqrt{-1}} \left(k_1 F_1 \frac{\partial F_2}{\partial \tau_1} - k_2 F_2 \frac{\partial F_1}{\partial \tau_1} \right) \in A_{k_1+k_2+1,1}$$

and

$$[F_1, F_2]_2 := \frac{-1}{4\pi^2} \left(k_1(k_1+1) F_1 \frac{\partial^2 F_2}{\partial \tau_1^2} - 2(k_1+1)(k_2+1) \frac{\partial F_1}{\partial \tau_1} \frac{\partial F_2}{\partial \tau_1} + k_2(k_2+1) F_2 \frac{\partial^2 F_1}{\partial \tau_1^2} \right) \in A_{k_1+k_2+2,2}.$$

We remark that there is a so-called "Jacobi identity":

$$k_1 F_1 [F_2, F_3]_1 + k_2 F_2 [F_3, F_1]_1 + k_3 F_3 [F_1, F_2]_1 = 0.$$