Estimate of the dimensions of mixed weight Hilbert modular forms

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To investigate Hilbert modular forms is one of the important interest on number theory. In fact, we know many properties about same weight Hilbert modular forms. But, about mixed weight Hilbert modular forms, we know only a little result, except some general theory.

In 2001, M.H.Lee [12] stated the explicit formula of the generalized Rankin-Cohen operator on mixed weight Hilbert modular forms. And recently, S. Hayashida and N.P. Skoruppa [13] investigated the structure theorem of Hilbert Jacobi forms, by using mixed weight Hilbert modular forms. They constructed differential operators from the coefficients of the Taylor expansion of Hilbert Jacobi forms to mixed weight Hilbert modular forms. This is an extension of the correspondence between the coefficients of the Taylor expansion of elliptic Jacobi forms to elliptic modular forms (cf. Eichler-Zagier [3]).

But the structure theorem of the space of mixed weight modular forms are unknown. The purpose of this exposition is to determine the structure of the space of mixed weight modular forms in some easy cases. To determine the structure, we estimate the dimensions of mixed weight Hilbert modular forms by means of differential operators, according to the exposition by Aoki [1].

1 Definitions and Notations

1.1 Hilbert modular forms

We denote the totally real quadratic field by K, its discriminant by d_K, its integer ring by $\mathcal{O}$, and its fundamental unit by ε. For $x = u + v\sqrt{d_K} \in K$, we write its conjugate number by $x' = u - v\sqrt{d_K} \in K$ and its trace by $\mathrm{tr}(x) = x + x' = 2u \in \mathbb{Q}$. We put the dual of $\mathcal{O}$ by

$$\mathcal{O}^* := \{\nu \in K \mid \mathrm{tr}(\mu\nu) \in \mathbb{Z} \text{ for any } \mu \in \mathcal{O}\}.$$

We denote the complex upper half plane by

$$\mathbb{H} := \{\tau \in \mathbb{C} \mid \mathrm{Im}(\tau) > 0\}.$$

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