We remark that this F is bounded at each cusps by Köcher principle. We denote by $A_{k,s}(\Gamma)$ the space of all Siegel modular forms of weight $\rho_{k,s}$ with respect to Γ. We remark $A_{k,0}(\Gamma) = A_k(\Gamma)$. It is easy to show that if s is odd and if $-E_4 \in \Gamma$, then $A_{k,s}(\Gamma) = \{0\}$. Put $A_{*,s}(\Gamma) := \bigoplus_{k\in\mathbb{Z}} A_{k,s}(\Gamma)$. The space $A_{*,s}(\Gamma)$ is a graded module of $A_*(\Gamma)$ or R, where R is a subring of $A_*(\Gamma)$ generated by the first four generators in **Theorem 1**.

The aim of this exposition is to determine the structure of $A_{*,2}(\Gamma)$ as a graded module of R. The structure of $A_{*,2}(\mathrm{Sp}(2,\mathbb{Z}))$ was already determined by Satoh [Sa] and Ibukiyama [Ib3]. There are ten generators, whose weights are

$$\begin{array}{llrl} 10 = 4+6, & 16 = 6+10, & & 21 = 4+6+10+1, \\ 14 = 4+10, & 18 = 6+12, & & 23 = 4+6+12+1, \\ 16 = 4+12, & 22 = 10+12, & & 27 = 4+10+12+1 \\ & & \text{and} & 29 = 6+10+12+1. \end{array}$$

To show this, they used the dimension formula of modular forms. In this exposition we will give this result by another way. By our way, we can determine the module structure of $A_{*,2}(\Gamma)$ for $\Gamma = \Gamma_0(2), \Gamma_{0,\psi_3}(3)$ or $\Gamma_{0,\psi_4}(4)$.

Theorem 2. *For each* $\Gamma = \mathrm{Sp}(2,\mathbb{Z}), \Gamma_0(2), \Gamma_{0,\psi_3}(3)$ *or* $\Gamma_{0,\psi_4}(4)$*, the graded module* $A_{*,2}(\Gamma)$ *is generated by ten modular forms.*

Γ	The weights of generators (Type 1)	The weights of generators (Type 2)	References
$\mathrm{Sp}(2,\mathbb{Z})$	$10, 14, 16, 16, 18, 22$	$21, 23, 27, 29$	Satoh [Sa] Ibukiyama [Ib3]
$\Gamma_0(2)$	$6, 6, 8, 8, 10, 10$	$11, 13, 13, 15$	Aoki [Ao]
$\Gamma_{0,\psi_3}(3)$	$4, 4, 5, 6, 7, 7$	$8, 9, 9, 11$	
$\Gamma_{0,\psi_4}(4)$	$3, 3, 4, 4, 5, 5$	$6, 7, 7, 8$	

We remark two points. The first point is, in all cases, these generators are obtained from the generators of R using by differential operators. Indeed, the first six generators are obtained from two generators of R using by Rankin-Cohen type differential operators in [Sa]. And the last four generators are obtained from two generators of R using by Rankin-Cohen-Ibukiyama type differential operators in [Ib3]. The second point is, in all cases, these modules are not free. There are relations called Jacobi Identities.