

Vector valued Siegel modular forms of degree 2 with small levels

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1 Introduction

On the structure theorem of Siegel modular forms of degree 2, Igusa [Ig1, Ig2] determined the structure of Siegel modular forms with respect to the full modular group $\mathrm{Sp}(2, \mathbb{Z})$. There are five generators of weight 4, 6, 10, 12 and 35. First four generators are algebraically independent and the square of the last generator is in the subring generated by first four.

Recently, Aoki and Ibukiyama [AI] indicated that the ring of Siegel modular forms with small level has similar structure. That is, on the ring of Siegel modular forms of degree 2 with respect to the congruent subgroup of level $N = 1, 2, 3, 4$ (for $N = 3, 4$, taking Neven-type case with character), there are five generators, among which four generators are algebraically independent and the square of the last generator is in the subring generated by first four.

On the structure of vector valued Siegel modular forms of degree 2 with respect to the symmetric tensor of degree 2, Satoh [Sa] and Ibukiyama [Ib3] determined the structure with respect to the full modular group. There are ten generators with some relations.

The original proofs of above structure theorems are various. However, now we can prove all of them by using the elementary estimation of the dimension of the space of Siegel modular forms. In this exposition, we study this method.

By this method, we also determined the structure of vector valued Siegel modular forms with small level. This structure is similar to the structure with respect to the full modular group.