Using Igusa's theorem it is a simple exercise to verify the following table

k	$k<20$	20	22	24	26	28	30	32	—
$\dim S_k^?(\Gamma_2)$	0	1	1	2	2	3	4	5	$= O(k^3)$

Apart from the last two listed problems it remains

$\boxed{\text{1) multiplicity one for } S_k^?(\Gamma_2)\ ?}$

Another open problem is the relation of the interesting Siegel Hef's and Galois-representations. If there would be such a relation — everybody believes, of course, that there is, then ~~the~~ the generalized Ramanujan-Petersson-conjecture should hold for Siegel-mod. forms:

$\boxed{4)\ F\ \text{Hef},\ \in S_k^?(\Gamma_2),\ F = \prod \frac{1}{Q_p(p^{-s})},\ \text{does hold:}\quad Q_p(p^{-s_0}) = 0 \overset{?}{\Longrightarrow} \operatorname{Re} s_0 = k-3/2}$

This is of course false for non-interesting Hef, but that is quite well understood since anything can be played back to the elliptic case there. There is another open problem, whose analogue for elliptic mod. forms is open too.

Let $\mathcal{H}_k(\Gamma_i) = \mathbb{Q}$-algebra generated by $T(n)$ $(n=1,2,\ldots)$ on $S_k(\Gamma_1)$ resp. $S_k^?(\Gamma_2)$

Clearly $\mathcal{H}_k(\Gamma_i) =$ finite direct sum of number fields

~~There is~~

Maeda (≈ 1990) states $\mathcal{H}_k(\Gamma_1) =$ field for $k = 12, \ldots$, seen. [illegible]

$\boxed{\text{5) Does } \mathcal{H}_k(\Gamma_2) \text{ split over } \mathbb{Q} \text{ or is it always equal to a \# field}}$