defines a Hecke-equivariant embedding

$$V: J_{k,1} \xrightarrow{\approx} \underbrace{M_k^*(\Gamma_2)}_{\text{Maaß-Spezialschaar}} \subseteq M_k(\Gamma_2)$$

On the other hand

Theorem

$$J_{k,1} \approx M_{2k-2}(\Gamma_1) \quad (\text{Hecke-equivalently}).$$

Thus

Theorem

$M_k^*(\Gamma_2) \underset{\text{equiv.}}{\overset{\text{Hecke}}{\approx}} M_{2k-2}(\Gamma_1)$; More precisely, If $F \in M_k^*(\Gamma_2)$ Hef, then ~~If $f \in M_{2k-2}(\Gamma_1)$ Hef,~~

~~F corresp. to f via the above isom., then~~

$$Z_F(s) = L_f(s)\, L_{E_2}(s-k+2)$$

with a suitable $f \in M_{2k-2}(\Gamma_1)$.

Jacobi forms are quite well understood now, their theory is complete now in a sense. So

Study Siegel mod forms via their Fourier Jac. development.

Another result related to this program

Theorem (Kohnen & N)

Let $F, G \in S_k(\Gamma_2)$, $F = \sum \phi_m q'^m$, $G = \sum \psi_m q'^m$, ~~that~~

$$D_{F,G}(s) = \zeta(2s-2k+4) \sum \frac{\langle \phi_m | \psi_m \rangle}{m^s} \quad (\langle\,,\rangle = \text{Petersson scalar product on } J_{k,m})$$

then

$D_{F,G}(s)$ can be merom. continued to $\mathbb{C}$ and satisfies the same functional equation as the Andrianov-zeta-function $Z_F(s)$ for $f \in M_k(\Gamma_2)$.