Theorem (Igusa $\approx$ 50's)

$M_4(\Gamma_2), M_6(\Gamma_2), S_{10}(\Gamma_2), S_{12}(\Gamma_2)$ are 1-dimensional.

$\left(S_k(\Gamma_2) = \{ F \in M_k(\Gamma_2) \mid a_F(Q) = 0 \text{ unless } Q > 0 \} \right)$.

Let $\psi_4, \psi_6, \chi_{10}, \chi_{12}$ non zero elements in these spaces

then

$$M_{2*}(\Gamma_2) = \bigoplus_{k \in 2\mathbb{Z}} M_{2k}(\Gamma_2) = \mathbb{C}[\psi_4, \psi_6, \chi_{10}, \chi_{12}].$$

Using this and some "tricks" there were tables produced

Resnikoff – Saldana : FC's for $\psi_4, \dots, \chi_{12}$ (Crelle, $\approx$ 72)

Kurokawa : using Res.-Sal. tables computed the Hef in $S_{10}, \dots, S_{20}$ ~~($\approx$ 1978)~~

except from one Hef in S_{20} all Hef had the property:

$a_F(Q)$ depends only on Q and the discriminant,

the eigenvalues are connected to eigenvalues of ellipt. mod. forms (Inventiones $\approx$ 78)

An explanation of this phenomenon was given by Maass and completed by Zagier: Here we have to introduce Jacobi forms, but only a special kind:

$F \in M_k(\Gamma_2)$, then $\quad F = \sum_{m \geq 0} \phi_m(\tau, z)\, q'^m \quad$ <u>Fourier – Jacobi – dev.</u> here

$$\phi_m \in J_{k,m} = \left\{ \phi : \mathfrak{H} \times \mathbb{C} \xrightarrow{\text{hol.}} \mathbb{C} \;\middle|\; \begin{array}{l} 1)\ \phi\left(\frac{-1}{\tau}, \frac{z}{\tau}\right) e\left(\frac{-2\pi i m z^2}{\tau}\right) \tau^{-k} = \phi(\tau) \\ 2)\ \phi = \sum_{\substack{\Delta, r \in \mathbb{Z} \\ \Delta \equiv r^2 (4m)}} c_\phi(\Delta, r)\, q^{\frac{r^2 - \Delta}{4m}} \zeta^r, \\ \quad \text{where } c_\phi(\Delta, r) \text{ depends only on } r \bmod 2m \\ \quad \text{and vanish } \nmid \Delta \geq 0. \end{array} \right\}$$

a/t par. ? equal var. will pr. 1

Theorem (Maass) The map

$$\phi = \sum_{\substack{\Delta, r \in \mathbb{Z} \\ \Delta < 0 \\ \Delta \equiv r^2 (4)}} c(\Delta)\, q^{\frac{r^2 - \Delta}{4}} \zeta^r \longrightarrow \sum_{\substack{Q \geq 0 \\ Q = [n, r, m]}} \sum_{a \mid \mathrm{cont}(Q)} a^{k-1} c_\phi\left(\frac{\mathrm{disc}\, Q}{a^2}\right) q^n \zeta^r q'^m$$

$$\left(a(0) = -\frac{2k}{B_{2k}} c_\phi(0) \right)$$