(6) D fund. discr., $h(D)$ = grp of $Sl_2(K)$-equiv. classes of binary pos. def. int. forms

χ char. of $h(D)$:

$$\sum_{Q \in h(D)} \chi(Q) \sum \frac{a_F(Q\ell)}{\ell^s} = \text{const.}\left(\sum \chi(Q) \sum \frac{r_Q(n)}{n^s}\right)^{-1} Z_F(s)$$

The proof of these facts is quite hard, in particular the proof of the mer. cont. and functional equation.

1) Open problem: Is a Hef F uniquely det. by its eigen-values ?

This is not known, so weaken the question:

Canonical subspaces such that multiplicity one holds on them ?

Yes, there are the results:

Theorem (Klingen) Let $f \in S_k(\Gamma_1)\left(= \{ f \in M_k(\Gamma_1) \mid a_f(0) = 0\}\right)$, set

$$kf = \sum_{A \in C_{2,1} \backslash \Gamma_2} \tilde{f}|_k A \qquad (\tilde{f}(\tau, z, \tau') = f(\tau))$$

The

$$f \to kf$$

defines Hecke-equiv. injection

$$S_k(\Gamma_1) \hookrightarrow M_k(\Gamma_2).$$

If $f \in S_k(\Gamma_1)$ is Hef the

$$Z_{kf}(s) = L_f(s) L_f(s-k+2).$$

Since then there was a long period where nothing was known. So there was the demand for examples. Thus one has to consider 2)