To 2)

There are several answers. But one which can immediately put on a computer is

$$M_*(\Gamma_1) = \bigoplus_{k\in\mathbb{Z}} M_k(\Gamma_1) = \mathbb{C}[E_4, E_6],$$

$$E_4 = 1 + 240 \sum_{n=1}^{\infty} \sigma_3(n) q^n$$

$$E_6 = 1 - 504 \sum_{n=1}^{\infty} \sigma_5(n) q^n .$$

Siegel mod. forms of genus 2

This is probably the most naive and simplest generalisation of the elliptic mod. forms.

Let

$$Q(x_1, \dots, x_r) \text{ pos. def., unimod. (as above)},$$

set

$$\Theta_Q^{(2)} = \sum_{n,r,m\in\mathbb{Z}} \#\left\{(\vec{x},\vec{y}) \in (\mathbb{Z}^r)^2 \;\middle|\; \begin{array}{l} n = Q(\vec{x}) \\ m = Q(\vec{y}) \\ r = Q(\vec{x}+\vec{y}) - Q(\vec{x}) - Q(\vec{y}) \end{array}\right\} q^n \zeta^r q'^m$$

$$\left(q = e^{2\pi i \tau},\ q' = e^{2\pi i \tau'},\ \zeta = e^{2\pi i z},\ \tau, \tau' \in \mathfrak{H},\ z \in \mathbb{C}\right)$$

then

$$\Theta_Q^{(2)} \in M_{r/2}(\Gamma_2).$$

$M_k(\Gamma_2)$ = Siegel mod. forms of weight $k\ (\in 2\mathbb{Z})$ on $\Gamma_2 = Sp(2,\mathbb{Z})$

$$= \left\{ F : \mathfrak{H}_2 \xrightarrow{\text{h.l.}} \mathbb{C} \;\middle|\; \begin{array}{l} \text{a) } F \text{ per. in each var. per. } 1 \\ \text{b) } F(\tau, z, \tau') = F(\tau', z, \tau), \\ \text{c) } \cancel{F\left(\frac{-1}{\tau}, \frac{z}{\tau}, \tau' - \frac{z^2}{\tau}\right) = \tau^k F(\tau, z, \tau')} \\ \quad F(\tau, z+\tau, \tau'+2z+\tau) = F(\tau, z, \tau') \end{array} \right\}$$

$$\left(\mathfrak{H}_2 = \{(\tau, z, \tau') \in \mathfrak{H}\times\mathbb{C}\times\mathfrak{H} \mid \operatorname{Im}\tau \operatorname{Im}\tau' - (\operatorname{Im} z)^2 > 0\}\right)$$

As in the elliptic case

(where $Z = \begin{pmatrix} \tau & z \\ z & \tau' \end{pmatrix}$)

$$\text{a), b), c)} \iff F|_k \begin{pmatrix} A & B \\ C & D \end{pmatrix}(Z) := F\left((AZ+B)(CZ+D)^{-1}\right) \cdot \det(CZ+D)^{-k} = F(Z) \text{ for all } \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \Gamma_2 .$$