Note that 1) is a sensible question even from the naive point of view, if you recall that arithmetically interesting number like representation numbers occur as FC's.

Satisfactory answers are known since a long time to both questions.

To 1)

Theorem (ess. Hecke)

On M_k there exists nat. sequence $T(n)$ $(n=1,2,3,\dots)$ of operators (Hecke-operators) such that:

(1) M_k contains a (finite) basis of simultaneous eigenforms for all $T(n)$ (Hecke-eigenforms)

(2) $f \in M_k$ HeF, $T(n) f = \lambda(n) f$, then $\lambda(n)$ = algebr. integer
$L_f(s)$ = L-series of f := $\sum \lambda(n) n^{-s}$ absol. conv. for $\mathrm{Re}\, s \gg 0$,
~~one has~~

(3) $L_f(s) = \prod_p \frac{1}{1-\lambda(p)p^{-s}+p^{k-1-2s}}$ (L_f has Euler product)

(4) $L_f(s)$ has ~~anal.~~ merom. cont. to $\mathbb{C}$

(5) $L^*(f,s) := (2\pi)^{-s}\Gamma(s)L_f(s) = L_f(k-s)\cdot(-1)^{k/2}$ (functional equation)

(6) Any HeF f is uniquely determined by its select eigenvalues $(\lambda(n))$ (multiplicity 1)
(in fact: $\lambda(n) = a_f(n)/a_f(1)$)

Moreover it is known:

Theorem ~~(Deligne 74)~~ To any HeF f there corresponds a Galois rep., s.that $X^2-\lambda(p)X+p^{k-1}$ = char. polyn. of p-th Frob.
These Galois-modules are in the range of the Weil-conj., in partic.

$$|\lambda(p)| \leq 2p^{\frac{k-1}{2}} \qquad \text{(Ramanujan-Pet.-conj.)}$$

(i.e. $1-\lambda(p)p^{-s_0}+p^{k-1-2s_0}=0 \Rightarrow \mathrm{Re}\, s_0 = \frac{k-1}{2}$)