discrete symm: $SL_2(\mathbb{Z}) \ltimes \mathbb{Z}^2 =: \mathcal{J}(\mathbb{Z})$

$$\mathcal{J}_{k,m} = \{ \phi(\tau,z) \mid \phi(\gamma\cdot(\tau,z))\,\mu(\gamma,\tau,z) = \phi \;\; \forall \gamma \in \mathcal{J}(\mathbb{Z})$$

+ "reg. conditions" $\}$.

Fact: Any $\phi \in \mathcal{J}_{k,m}$ can be written as

$$\phi = \sum_{\substack{n,r\in\mathbb{Z} \\ 4mn-r^2\geq 0}} c(n,r)\, e^{2\pi i(n\tau + rz)}$$

Here you see again a sequence $c(n,r)$ indexed by integers n,r satisfying $4mn - r^2 \geq 0$.

Ex ~~1) almost a Jacobi form~~ meromorphic examples

1.a) modular forms

2.a) "meromorphic" Jacobi, ~~f~~

$$\wp(\tau,z) = \frac{1}{z^2} + \sum_{\substack{\lambda\in\mathbb{Z}\tau+\mathbb{Z} \\ \lambda\neq 0}} \left(\frac{1}{(z+\lambda)^2} - \frac{1}{\lambda^2} \right) = \text{Weierstr. fct. assoc. to } \mathbb{Z}\tau+\mathbb{Z}.$$

2.) F as above, $F = F^{-1}$, $x_0 \in \mathbb{Z}^n$ fixed

$$\vartheta_F(\tau,z) := \sum_{x\in\mathbb{Z}^n} e^{2\pi i\left(\tau\frac{1}{2}x^tFx + z\,x^tFx_0\right)} \in \mathcal{J}_{\frac{n}{2},\,\frac{1}{2}x_0^tFx_0}.$$