and preserves Petersson scalar products.

One can prove even more, namely, the above map defines indeed an injection into

$$M^{D}_{k-1/2}(\quad) = \{ h \in M_{k-1/2}(\Gamma_0(\,--\,), \chi) \mid h = \sum_{N \equiv -D \bmod 4} c(N)\, q^{N} \},$$

This map is surjective if $F^2 \mid m$.