Let $J_{k,m}^{f,d}$ the subspace of $J_{k,m}$ corresponding to Θ_{m/d^2}^{f}.

By a closer study of these spaces one can find a more nice description of these $J_{k,m}^{f,d}$.
Namely, there is a simple map

$$U_\ell : J_{k,m} \to J_{k,m\ell^2}$$
$$\phi \to \phi | U_\ell (\tau, z) = \phi(\tau, \ell z)$$

One can define notion of the space of Newforms $J_{k,m}^{1} \subseteq J_{k,m}$ with respect to the operators U_d, s.t. that

$$J_{k,m} = \bigoplus_{d^2 | m} J_{k,m/d^2}^{1} | U_d .$$

Furthermore, there is for each $p|m$ an involution W_p on $J_{k,m}$:

$$\phi | W_p = p^{-\alpha} \sum_{x \bmod p^\alpha} \phi|_{k,m} [\tfrac{x}{p^\alpha}] \quad (p^\alpha \| m)$$

The W_p commute and preserve the space $J_{k,m}^{1}$.
It turns out that

<u>Prop</u>

i) $J_{k,m}^{1,f} = \left\{ \phi \in J_{k,m}^{1} \;\middle|\; \phi | W_p = \begin{cases} \phi & p \nmid f \\ -\phi & p | f \end{cases} \forall p \right\}$

ii) $J_{k,m}^{d,f} = J_{k,m/d^2}^{1,f} | U_d$.

Now let me turn back to the problem of connecting Jacobi-forms and modular forms.
Because of the description of the $J_{k,m}^{f,d}$ sketched / given above it is enough to consider the spaces $J_{k,m}^{1,f}$.

And here one has finally

<u>Theorem</u> Let $k, m \in \mathbb{N}$.
Let $F | 2m$, χ a primitive Dirichlet character mod F, s.t. $\chi(-1) = (-1)^k$
let Q the greatest natural number s.t. $Q^2 | m$,
let $f = \prod_{\chi_p(-1) = -1} p$ where $\chi = \prod_{p|F} \chi_p$ is the natural decomposition of χ_p.
Then the map

$$\phi = \sum c(n,r) q^n \zeta^r \longrightarrow \sum_{N \geq 0} \Big[\sum_{\substack{\rho (\bmod 2m) \\ N \equiv -\rho^2 \bmod 4m \\ (\rho, Q) = 1}} \chi(\rho)\, c\big(\tfrac{N+\rho^2}{4m}, \rho\big) \Big] q^N$$

defines an injection

$$J_{k,m}^{1,f} \hookrightarrow M_{k-1/2}(\Gamma_0(4[m, F^2]), \chi),$$

(least common multiple)

This map has the following properties:
cusp forms $\mapsto$ cusp forms, Eisenstein series $\mapsto$ Eisenstein series, it is compatible with Hecke-operators