⑤ Find suitable $\Gamma_0^0(n,n'), \chi$, i.e. ~~they~~ shall be of the kind that

(*) $\quad (Th_n^{(i)})^* \underset{\text{as } \tilde\Gamma\text{-modules}}{\hookrightarrow} (V_k^{\tilde\Gamma})$ with multiplicity one

($\underset{\text{Frob. recipr.}}{\Leftrightarrow}$ k appears in the restriction of the character of $(Th_n^{(i)})^*$ with multiplicity one).

o.k. That I want to have "multiplicity one" comes from the fact, that ~~I want to~~ ~~hope~~ in this case $(Th_n^{(i)})^*$ will be a component factor of $V_k^{\tilde\Gamma}$ (no choice for the embedding (*) up to constants), so one can expect that the subspace ~~corresponding to~~ ~~the $V_k^{\tilde\Gamma} \cong Th_n^{(i)}$~~

$$M^{(i)}_{k-1/2}(\Gamma_0^0(n,n'),\chi) \text{ corresponding to } Hom_{\tilde\Gamma}((Th_n^{(i)})^*, M_{k-1/2}) \hookrightarrow Hom^{(i)}_{\tilde\Gamma}(V_k^{\tilde\Gamma}, M_{k-1/2})$$

is something canonical.

⑥ Deduce that

$$J_{k,m}^{(i)} \xrightarrow{\cong} M^{(i)}_{k-1/2}(\Gamma_0^0(n,n'),\chi).$$

Note that this isomorphism is uniquely (up to normalization) ~~with in the above construction~~. determined by the above construction.

Now that it is clear what kind of computations one has to do let me state the results.

Proposition: (i) There is a sequence ~~Θ_m^f~~ of irreducible pairwise distinct characters Θ_n^f of $\tilde\Gamma$, where $n \in \mathbb{N}$, $f|n$, f squarefree, such that for all m:

$$(\text{character of } Th_m^*) = \sum_{\substack{fd^2|m \\ f \text{ squarefree}}} \Theta^f_{m/d^2}$$

(ii)