Using this and $\phi|_{k,m} A = \phi$ $(\forall A \in \Gamma)$ it is easy to verify that for $\phi = \sum h_g \vartheta_{m,g}$ the h_g transform like

$$(h_1, \ldots, h_{2m})|_{k-1/2}\, \alpha = (h_1, \ldots, h_{2m})\, R(\alpha)^{-1} \quad \forall \alpha \in \tilde{\Gamma}.$$

From this it is clear that the h_g are modular forms of weight $k-1/2$,

$$h_g \in M_{k-1/2} \quad \Big(= \bigcup_{n \geq 1} M_{k-1/2}(\Gamma(4n))\Big),$$

to be more precise, a careful analysis of the representation R shows:

$$h_g \in M_{k-1/2}(\Gamma(4m)).$$

~~So we have~~ Reversing these arguments one gets

$$J_{k,m} \simeq \{(\ldots, h_g, \ldots) \in M_{k-1/2}^{(2m)} \mid (\ldots, h_g, \ldots)|_{k-1/2}\, \alpha = (\ldots, h_g, \ldots) R(\alpha) \ \forall \alpha \in \tilde{\Gamma}\}.$$

Using this Eichler proved: ~~that~~

<u>Theorem</u> Let k even. The map $\phi = h_0 \vartheta_{1,0} + h_1 \vartheta_{1,1}$ ~~defines~~ $\longrightarrow h_0(4\tau) + h_1(4\tau)$ defines an isomorphism $J_{k,1} \xrightarrow{\simeq} M_{k-1/2}^{+} := \{h \in M_{k-1/2}(\Gamma_0(4)) \mid h = \sum_{N \equiv 0, -1\,(4)} c(N) q^N\}$

This isomorphism has several properties (Hecke-equivariant) and plays a role in the proof of the Saito-Kurokawa conjecture. But these are not the points that I will consider here. In any case it may be of interest to generalize this theorem. So I come now to the proper kernel of this talk.

What one wants to do: find a map of the form

$$J_{k,m} \ni \phi \simeq (h_1, \ldots, h_{2m}) \longrightarrow \text{linear combination of the } h_g$$

which will map isomorphically

$$J_{k,m} \longrightarrow \text{nice subspace of } M_{k-1/2}(\Gamma_0(n), \chi) \text{ for some } n$$
$$\big(\text{or at least: } M_{k-1/2}(\Gamma_0^0(n,n'), \chi)\big).$$

Well, such a map ~~does~~ not exist in general.

To explain ~~why this~~ the reason for this, and also in order to explain what one can do else, let me rewrite the whole problem more abstractly: