Beweis von Satz 2

$$S := \vartheta_{v+r} \mid A = \sum_{v' \in v+r} e^{\pi i\, tr\left(\frac{a}{c} - \frac{1}{c(cz+d)}\, v'^2\right)}$$

$$= \sum_{\substack{\tilde v \in \tilde r/cr \\ \tilde v \equiv v\,(r)}} e^{\pi i\, tr\, \frac{a}{c} \tilde v^2} \sum_{v' \in \tilde v + cr} e^{\pi i\, tr\left(\frac{-1}{c(cz+d)}\, v'^2\right)}$$

denn ist $v' \in \tilde v + cr$, etwa $v' = \tilde v + cw$, so $e^{\pi i\, tr\, \frac{a}{c}(\tilde v + cw)^2} = e^{\pi i\, tr\, \frac{a}{c} \tilde v^2}\, e^{2\pi i\, tr\, a \tilde v w}\, e^{\pi i\, tr\, a c w^2}$
$= e^{\pi i\, tr\, \frac{a}{c} \tilde v^2}$.

Sonst haben wir weiter

$$S = \sum_{\substack{\tilde v \in \tilde r/cr \\ \tilde v \equiv v(r)}} e^{\pi i\, tr\, \frac{a}{c} \tilde v^2} \quad \text{(*)}\; n\left(\frac{z}{i}\right)^{\nu/2} \mathcal{D}(r)^{-1/2} \sum_{w \in \widetilde{cr}/cr} e^{2\pi i\, tr\, \tilde v \cdot w}\, \vartheta_{w+cr}(c(cz+d))$$

hier ist $\widetilde{cr} = \{ y \in k \mid tr\, y \cdot cr \subseteq \mathbb{Z} \} = \{ y \in k \mid cy \in \tilde r \} = \frac{1}{c} \tilde r$,

durch ~~(cw → w)~~

$$S = \sum_{\substack{\tilde v \in \tilde r/cr \\ \tilde v \equiv v(r)}} n\left(\frac{c(cz+d)}{i}\right)^{\nu/2} \mathcal{D}(cr)^{-1/2} \sum_{w \in \frac{1}{c} \tilde r/cr} e^{\pi i\left(tr\left(\frac{a}{c} \tilde v^2 + 2 \tilde v w\right)\right)}\, \vartheta_{w + cr}(c(cz+d))$$

$$= n\left(\frac{c(cz+d)}{i}\right)^{\nu/2} \mathcal{D}(cr)^{-1/2} \sum_{\substack{\tilde v \in \tilde r/cr \\ \tilde v \equiv v(r)}} \sum_{w \in \frac{1}{c} \tilde r/cr} e^{\pi i\, tr\left(\frac{a}{c} \tilde v^2 + 2 \tilde v w + cd(w)^2\right)}\, \vartheta_{cw + c^2 r}(z)$$

$$= n\left(\frac{c(cz+d)}{i}\right)^{\nu/2} \mathcal{D}(cr)^{-1/2} \sum_{\substack{\tilde v \in \tilde r/cr \\ \tilde v \equiv v(r)}} \sum_{w \in \tilde r/c^2 r} e^{\pi i\, tr\left(\frac{a}{c} \tilde v^2 + 2 \frac{\tilde v \cdot w}{c} + \frac{d}{c} w^2\right)}\, \vartheta_{w + c^2 r}(z)$$

$$tr\left(\frac{a}{c} \tilde v^2 + 2 \frac{\tilde v \cdot w}{c} + \frac{d}{c} w^2\right) = tr\left(\frac{a}{c}(\tilde v + dw)^2 - 2b\, \tilde v \cdot w - bd\, w^2\right)$$

$$= n\left(\frac{c(cz+d)}{i}\right)^{\nu/2} \mathcal{D}(cr)^{-1/2} \sum_{w \in \tilde r/r} e^{\pi i\, tr(-2b\, \tilde v \cdot w - bd w^2)} \sum_{\substack{\tilde v \in \tilde r/cr \\ \tilde v \equiv v + dw\,(r)}} e^{\pi i\, tr\, \frac{a}{c} \tilde v^2}\, \vartheta_{w+r}(z)$$

(*) Man beachte: $tr\, r \cdot r \subseteq \mathbb{Z}$, $tr\, \mathcal{O} \frac{v^2}{2} \subseteq \mathbb{Z}\ \forall v \in r$, d.h. $\left(\frac{v^2}{2} \mid v \in r\right) \subseteq \mathcal{D}^{-1}$.