Theorem. *Let*

$$R(X) = \frac{X/2 - \exp(X)}{\exp(X) - 1} + \frac{1}{X}.$$

Then

$$\zeta_{\Delta,n}(0) = \text{–}\textit{coefficient of } X^n \textit{ in } (n+1)\frac{X^{n+1}}{(1-\exp(-X))^{n+1}}R(X).$$

$$\begin{aligned} R(X) = & \frac{-1}{3}X + \frac{1}{24}X^2 + \frac{1}{720}X^3 - \frac{1}{1440}X^4 - \frac{1}{30240}X^5 + \frac{1}{60480}X^6 + \frac{1}{1209600}X^7 \\ & - \frac{1}{2419200}X^8 - \frac{1}{47900160}X^9 + \frac{1}{95800320}X^{10} \\ & + \frac{691}{1307674368000}X^{11} - \frac{691}{2615348736000}X^{12} - \frac{1}{74724249600}X^{13} \\ & + \frac{1}{149448499200}X^{14} + \frac{3617}{10670622842880000}X^{15} - \frac{3617}{21341245685760000}X^{16} \\ & - \frac{43867}{5109094217170944000}X^{17} + \frac{43867}{10218188434341888000}X^{18} \\ & + \frac{174611}{802857662698291200000}X^{19} - \frac{174611}{1605715325396582400000}X^{20} \\ & - \frac{77683}{1410110003939180544000O}X^{21} + \frac{77683}{2820220007878361088000O}X^{22} + O(X^{23}) \end{aligned}$$