Theorem. *Let*

$$\zeta_{\Delta,n}(s) = \sum_{\substack{q \geq 1 \\ k \geq q}} (-1)^{q+1} \frac{d_{n,q}(k)}{(k(k+n+1-q))^s},$$

where

$$d_{n,q}(X) = q \binom{n}{q} \binom{X+n}{n} \binom{X+n-q}{n} \left[\frac{1}{X} + \frac{1}{X+n+1-q}\right].$$

Then

$$\begin{aligned} \sum_{n=0}^{\infty} \zeta_{\Delta,n}(0)\, T^n &= -\frac{d}{dT}\left(\frac{1}{2}\log(1-T) + \frac{1}{1-T} + \frac{T}{(1-T)\log(1-T)}\right) \\ &= -\frac{2}{3}T - \frac{11}{8}T^2 - \frac{379}{180}T^3 + O(T^4) \end{aligned}$$

$-\zeta_{\Delta,n}(0)$	0	10	20
1	$\frac{2}{3}$	$\frac{1802771617643}{217945728000}$	$\frac{659591781989699265581 3}{40481626907344896000 0}$
2	$\frac{11}{8}$	$\frac{3646774870379}{402361344000}$	$\frac{1258480810797909956685851}{73570956727261593600000}$
3	$\frac{379}{180}$	$\frac{3157021764343}{320246784000}$	$\frac{37939749705011347127672803229}{2117280170914679586816000000}$
4	$\frac{821}{288}$	$\frac{349503911}{32800768}$	$\frac{12185000243142424571383754 9}{65042846850498956906987 52}$
5	$\frac{36353}{10080}$	$\frac{22918508433586751}{2000741783040000}$	$\frac{3788307486100939477875403 3}{19377798550373007360000 00}$
6	$\frac{75521}{17280}$	$\frac{4196410148598557}{342372925440000}$	$\frac{1198406759999188304392379}{58841149718134784000000}$
7	$\frac{2331983}{453600}$	$\frac{37071282097563643441}{2838385676206080000}$	$\frac{114681258367734471893263230238403}{5413323669636552217067520000000}$
8	$\frac{530183}{89600}$	$\frac{74573503887825627697}{5377993912811520000}$	$\frac{1900968602085488346909459612187}{86391030350426595655680000000}$
9	$\frac{64168051}{9580032}$	$\frac{247401847890274420402 13}{168600109166641152000 0}$	$\frac{3557295558691126964653771512330110 53}{1558544187991877063298137456640000 0}$
10	$\frac{130342067}{17418240}$	$\frac{154663215226474117}{9989239603200000}$	$\frac{92726489852828908926057231366787436 89}{39214982794634326108791845683200000 0}$