1. $H(f^*(\mathcal{E})) = f^* H(\mathcal{E})$,

2. Given any exact sequence

$$0 \to \mathcal{E}_1 \to \mathcal{E}_2 \to \mathcal{E}_3 \to 0$$

of v. b. on V,

$$H(\mathcal{E}_2) = H(\mathcal{E}_1) + H(\mathcal{E}_3).$$

3. When L is an inv. sheaf on V with

$$x = c_1(L) \in H^2(V),$$

$$H(L) := \sum_{\substack{m \text{ odd} \\ m \geq 1}} \left[2\zeta'(-m) + \zeta(-m)\left(\sum_{j=1}^{m} \frac{1}{j} (1 - \tfrac{1}{2}\gamma) \ln^2 \right) \right] \frac{x^m}{m!} - \frac{x}{4} \ln^2.$$

VIII. Proof [11].

the key point is to calculate

$$\sum_{q=0}^{n} H_{1}{}^{q} \zeta_q(0).$$

Then the problem becomes to calculate

$$\zeta_n(0) := \sum_{\substack{k \geq q \\ n \geq q \geq 1}} (-1)^q \frac{d_{n,q}(k)}{(k(k+1)+1-q))^s} \Big|_{s=0}.$$

Lemma (N. Skoruppa). For any $a \geq 0$, any polynomial $P(x) = \sum_{i=0}^{N} c_i x^i$

$$\sum_{k=1}^{\infty} P(k) (k(k+a))^{-s} \Big|_{s=0} = \zeta P - P^*(-a)$$

where

$$\zeta P := \sum_{i=0}^{N} c_i \zeta(-i)$$

$$P^*(x) = \sum_{i=0}^{N} c_i \frac{x^{i+1}}{i+1}.$$