II. Example [10]

$X = \mathbb{P}^1_{\mathbb{Z}}$, $Y = \operatorname{Spec}\mathbb{Z}$. $E = \mathcal{O}_{\mathbb{P}^1_{\mathbb{Z}}}$. Consider the Fubini-Study metric on $\mathbb{P}^1_{(\mathbb{C})}$. We can easily find that

$$a(0) = 48\zeta'(-1) - 2 - 8\log 2.$$

III In fact, the only problem is to find the following fact [2].

$$\zeta_E(0) = -\frac{r(E)}{3}(g-1) + \frac{1}{2}d(E) - h^0(E).$$

III. Arithmetic Riemann-Roch-Hirzebruch theorem for Relative Dimension 1. [11].

Let $f: X \to Y$ be a projective, generic smooth morphism between two arithmetic varieties with relative dimension 1. Suppose that the relative tangent sheaf $T_{X/Y}$ is equipped with an Hermitian metric, which induces Kähler metric on the fiber at infinity. Then for any Hermitian vector bundle (E, h) on X,

$$\hat{c}_1(\lambda(E), h_Q) = f_*(T(E, h))^{(1)} - a((1-g)(4\zeta'(-1) - \frac{1}{6} - \frac{2}{3}\log 2)\, r(E))$$

holds in $\widehat{CH}(Y)_{\mathbb{Q}}$.

IV. $a(g)$

From above, we have

$$a(g) = (1-g)\, a(0).$$