Constant Associated with Deligne's Riemann-Roch Theorem

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In this paper, we will determinate the constant associated with Deligne's Riemann-Roch theorem. During this process, we also formulate the arithmetic Riemann-Roch-Hirzebruch theorem. As less of space, we freely use the "standard" notation without explanation.

I. Deligne's Riemann-Roch Theorem [3],[7].

Let k be a number field with O_k as its ring of integers. Let $f: X \to Y = \operatorname{Spec}(O_k)$ be a semistable arithmetic surface. Then for any Hermitian vector bundle (E,h), we have

1. on Y, there exists a canonical algebraic isomorphism

$$\lambda(E)^{12} \simeq O(\Delta)^{r(E)} < W_{X/Y}, W_{X/Y} >^{r(E)} <\det E, \det E \, W_{X/Y}^{-1} >^6 IG(E)^{-12};$$

2. If $\lambda(E)$ is with Quillen metric, the above isomorphism becomes an isometry if the natural metric on the right hand side is timed by

$$\exp(r(E) a(g)),$$

where $a(g)$ is a constant only depends on the genus g