

and

$$F(x) = \left(-\frac{1}{1-e^{-x}} + \frac{1}{x} + \frac{1}{2} \frac{x e^{-x}}{1-e^{-x}} \right) \cdot \log 2$$

i.e.

$$= \left(-\frac{e^x}{e^x-1} + \frac{1}{x} + \frac{1}{2} \frac{x e^{-x}}{e^x-1} \right) \cdot \log 2$$

~~$$F(x) = \log 2 \cdot \sum_{r=0}^{\infty} \left(\frac{g(-r)}{r!} \right)$$~~

$$F(x) = \left(-\frac{1}{3} x + \sum_{\substack{r \text{ odd} \\ r \geq 3}} \frac{g(-r)}{r!} \frac{x^r}{r} + \sum_{\substack{r \text{ even} \\ r \geq 2}} \left(\frac{-\frac{1}{2} g(1-r)}{(r-1)!} \right) \frac{x^r}{r} \right) \cdot \log 2$$