but for $l = r+1$

$$\zeta(2s+l-r) = \frac{1}{2s} + O(1) \quad \text{for } s \to 0$$

$$\binom{-s}{l} = \frac{-s}{r+1}\binom{-s-1}{r} = \frac{-s}{r+1}\left((-1)^r + O(s)\right) \quad \text{for } s \to 0$$

thus

$$Z(0) = \sum_{1 \le \lambda \le a} D(\lambda) + \sum_r c_r \Big(\zeta(-r) - \sum_{1 \le \lambda \le a} \lambda^r\Big)$$

$$+ \frac{1}{2}\sum_r c_r \frac{(-a)^{r+1}}{r+1} \qquad \square$$

Thus we get

$$q\,\zeta_q(0) = \sum_{1} d_{n,q}(X)\Big|_{X^r \mapsto \zeta(-r)} + \frac{1}{2}\int_{-(n+1-q)}^{0} d_{n,q}(x)\,dx$$

This can be simplified, since the $d_{n,q}(X)$ are really nice polynomials:

__Lemma__

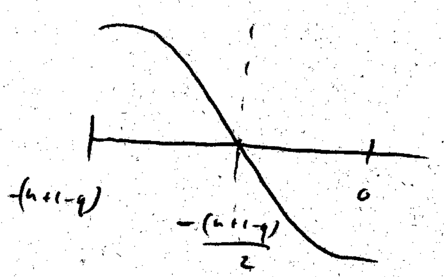
$$\cancel{d_{n,q}(X-(n+1-q)) = -d_{n,q}(X)}$$

__Lemma__ $d_{n,q}(-(n+1-q)-X) = -d_{n,q}(X)$

__Corollary__

$$\int_{-(n+1-q)}^{0} d_{n,q}(x)\,dx = 0$$

proof of Corollary: by the Lemma $d_{n,q}$ looks like




hence the integral vanishes.