(2)

considering for end in the situation

$$X \xrightarrow{f} Y \quad = \quad \mathbb{P}^n_{\mathbb{Z}} \to \operatorname{Spec} \mathbb{Z}$$

$$E \quad = \mathcal{O} = \mathcal{O}_{\mathbb{P}^n_{\mathbb{Z}}} = \text{structure sheaf}$$

$$h \quad = \text{trivial metric}$$

$$h_{X/Y} \quad = \text{Fubini–Study metric } h_{\mathbb{P}^n}$$

they gets by this (an equation in real numbers via a what gives $\widehat{CH}^1(\operatorname{Spec}\mathbb{Z})_{\mathbb{Q}} = \mathbb{R}$

$$\hat{c}_1(\lambda(\mathcal{O}), h_Q) = f_*\left(\widehat{Td}(T_{\mathbb{P}^n}, h_{\mathbb{P}^n})(1 + a\,R(T_{\mathbb{P}^n}))\right)^{(1)}$$

and they computed

$$\hat{c}_1(\lambda(\mathcal{O}), h_Q) = -\log n! + \sum_{q \geq 0} q(-1)^{q+1} \zeta_q'(0)$$

$$\zeta_q(s) = \operatorname{tr}'(\Delta^{-s}, A^{0,q}(\mathbb{P}^n)) \qquad (20)$$

$$f_*\left(\widehat{Td}(T_{\mathbb{P}^n}, h_{\mathbb{P}^n})\; a\,R(T_{\mathbb{P}^n})\right)^{(1)}$$

Lemma 2.2.1

$$= \text{coefficient of } X^n \text{ in } (n+1)\left(\frac{X}{1-e^{-X}}\right)^{n+1} R(X)$$

From this they computed

$$R(x) = \sum_{\substack{m \text{ odd} \\ m \geq 1}} \left[2\zeta'(-m) + \zeta(-m)\left(1 + \frac{1}{2} + \dots + \frac{1}{m}\right)\right]\frac{x^m}{m!}$$

Point is that this has to be corrected:

G-S. doubt Δ to define h_Q, but then h_Q is not a smooth metric

, one has to take

$$\square = \bar\partial\bar\partial^* + \bar\partial^*\bar\partial = \frac{1}{2}\Delta$$

Thus we have in the above to replace

$$\zeta_q(s) \quad \text{by} \quad 2^s \zeta_q(s)$$

$$\zeta_q'(0) \quad \text{by} \quad \log 2\, \zeta_q(0) + \zeta_q'(0)$$

hence

$$R(X)^{\text{corrected}} = R(X) + F(X) \quad \text{where}$$