Gillet-Soulé conjecture(d) — Yves André's Riemann-Roch seminar — 20 Nov. 90

$$(*)\ \begin{cases} f: X \to Y & \text{smooth, proj. morph. of arithmetic varieties} \\ (E,h) & \text{hermitian vbundle over } X, \text{ then} \\ \hat{c}_1(\lambda(E), h_Q) = f_*\big(\hat{ch}(E,h).\{\hat{Td}(\nabla_{X/Y}, h_{X,Y}) \\ \qquad (1 + a(R(T_{X/Y}))\}\big)^{(1)} \end{cases}$$

two things catch immediately our attention, namely those ingredients which make this look different from the usual ~~G-R-R-T~~ Grothendieck-RR-Theorem of algebraic geometry:

h_Q = Quillen-metric,

$R(E)$ certain genus of holomorphic vector bundles on complex manifolds,

given by a power series $R(x) \in \mathbb{R}[[x]]$

(i.e. $R(\text{line-bundle}) = R(c_1(\text{line bundle}))$,
$0 \to A \to B \to C \to 0$ exact sequence of v.b. then $R(B) = R(A) + R(C)$
$R(f^*A) = f^*R(A)$)

a is a canonical map: $H(X)$ = cplx cohomology of $X \to \hat{Ch}(X)_Q$.

$h_{X/Y}$ = any hermitian metric such that its restriction to X_y (fibres) is Kähler,

$\lambda(E)_Q$ = holomorphic line bundle on Y with

$$\lambda(E)_y = \bigotimes_q \Lambda^{max} H^{0,q}(X_y, E)^{(-1)^q} \quad \leftarrow \text{harmonic } 0,q\text{-forms}$$

$$h_{Q,y} = h_{L^2} \cdot \exp\Big(+\sum_q (-1)^{q+1} q \frac{d}{ds} \mathrm{tr}(\Delta^{-s}, A^{0,q}(X_y, E))\big|_{s=0}\Big)$$

$\uparrow$ L^2-norm induced by $h_{X/Y}$ and h on $0,q$-forms

What Gillet Soulé in "Analytic torsion and ..." actually did is to conjecture that

$\exists R$ such that $(*)$ holds true.

They computed R by