(8)

To construct elements of $M_n(\varrho)$ for a given ϱ the following observation is useful:

Theorem [Kloosterman, Wolfart-Nobs]:
Every irred. right rep. of Γ whose kernel is a cong. subgroup is obtained as subrep. of a "proper Weil-representation".

Weil representation associated to (M, Q)
- M finite abelian group
- $Q : M \to \mathbb{Q}/\mathbb{Z}$ quadratic form (i.e. $B(x,y) = Q(x+y) - Q(x) - Q(y)$ bilin. + $Q(x) = Q(-x)$)

~~Γ acts on $\mathbb{C}^M$:~~ Weil repr. $\omega = \omega_{(M,Q)}$ of Γ on $\mathbb{C}^M$:

$(f|\omega(T))(x) = e^{2\pi i Q(x)} f(x)$

$f|\omega(S) =$ finite Fourier transform of f w.r.t. $e^{2\pi i B(x,y)} = \gamma(1)^{-1}\sum_{y \in M} e(-B(x,y)) f(y)$

Given $\varrho : \Gamma \to GL(n, \mathbb{C})$ (irreducible ...) such that $((z,A) \mapsto z\,\varrho(A)^t) \leq \omega(M,Q)$

choose ~~$\varrho \leq$ Rep. of Γ~~ Weil representation a.t. (M, Q),
~~compute $\phi : M \to \mathbb{C}^n$ s.t. $\phi|A = \varrho(A)\phi$~~
choose even integral lattice $L \leq \mathbb{R}^{2k}$ s.t. $(L^*/L, \frac{x^2}{2} \bmod \mathbb{Z}) \cong (M, Q)$
choose spherical poly. p (i.e. homog. + $\Delta p \equiv 0$)

Theorem [everybody since 19th cent]

$$\mathbb{C}^M \ni f \longmapsto \Theta_f = \sum_{x \in L^*} p(x) f(x)\, q^{x^2/2} \quad \text{is a } \Gamma\text{-homomorphism}$$

(i.e. $\Theta_f |_{r+\deg p} A = \Theta_{f|A}$)

Corollary: $\sum_{x \in L^*} \phi(x) p(x)\, q^{x^2/2} \in M_{r+\deg p}(\varrho)$.

Choose subrep. of $\omega(M,Q)$ isom. to $(z,A) \mapsto z\cdot\varrho(A)^t$. Then $\exists$ basis f_j of subrep. such that $\phi|\omega(A) = \varrho(A)\phi$ where $\phi = \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}$.

$\gamma(a) = \sum_{x \in M} e(a\,Q(x))$.

Lemma: ω is proper repres. (not only proj.) iff $\gamma(a) \in \{\pm 1\}$ for all $\gcd(a, l) = 1$ with l = smallest int. > 0 such that $l\,Q(M) = 0$ (level of Q).