- one would like to see them because there is hope that these are interesting for their own sake
- shall show later how to construct $M_n(g)$ – at least for those g which are interesting here

we did this and calculated the $\varrho_{c,h}$

then we looked for the product expansion of these

__Lemma__ Let $f: \mathfrak{H} \to \mathbb{C}$ meromorphic and periodic w.p. 1. Then there exist uniquely ~~integers~~ $a_1, a_2, \ldots \in \mathbb{C}$ and ~~that~~ $r > 0$ and $N \in \mathbb{Z}$ s.t.

$$f = q^N \prod_1^{\infty} (1 - q^n)^{a_n} \qquad \text{for } |q| < r.$$

In general, if f is mod. function with zero or pole in $\mathfrak{H}$ then a_n wildly increasing

However, as we saw the $\varrho_{c,h}$ had few poles – just sufficiently many to exist. So one might expect that they also have few zeros, maybe no zeros in $\mathfrak{H}$, and then the a_n should be nice.

Indeed they have nice product expansions.

– transparency –

Indeed there are well-known functions with nice $\prod$-expansions:

$$\varphi_r = \frac{\eta(q^l)}{\eta(q)\ [r]_l} \qquad \begin{pmatrix} 1 \le r \le \frac{l-1}{2} \\ l = 2k+1 \text{ odd} \end{pmatrix} \quad \cancel{0 < r \le k}$$

These are also characters of some W-algebras (could not look up)

And indeed these functions even have a nice series expansion:

generalised Rogers-Ramanujan (Andrews-Gordon) identities

$$= \sum_{m_1, \ldots, m_{k-1} \ge 0} \frac{q^{A[\vec m] + b_r^t \vec m}}{(q)_{m_1} (q)_{m_2} \cdots (q)_{m_{k-1}}} \qquad A = \left(\min(i,j)\right)_{1 \le i,j \le k-1}$$

$$(b_r)_i = \min(i - r + 1, 0)$$