②

rational model of W

finitely many lowest-weight rep. $\omega_0, \omega_1, \dots, \omega_n$ of W (ω_0 = vacuum rep.)
such that

- $\chi_{\omega_\nu}(\tau)$ converging for $\tau \in \mathfrak{H} = \{\mathrm{Im}\,\tau > 0\}$, $q = e^{2\pi i \tau}$
- $\mathrm{span}\{\chi_{\omega_\nu}(\tau) \mid \nu \in 0, \dots, n\}$ is invariant under the action
  $(f, (\begin{smallmatrix} a & b \\ c & d \end{smallmatrix})) \longmapsto f(\frac{a\tau+b}{c\tau+d})$ of $\Gamma = SL_2(\mathbb{Z})$ on functions on $\mathfrak{H}$

Nahm: chiral symmetry algebra of a CQFT on torus with only finitely many lowest wt. rep. yields a rational model of a W-algebra

---

Consider rational model with central charge c and H_c = (family) set of conformal dim.
We have

$$H_c \ni h \longmapsto \xi_h = \text{conformal character assoc. to rep. with conf-dim } h$$

they satisfy

(1) $\xi_h \in \mathrm{Hol}(\mathfrak{H})$, $\xi_h \not\equiv 0$

(2) $\mathrm{span}\{\xi_h \mid h \in H\}$ is inv. under Γ w.r.t. action
$(\xi, A) \longmapsto \xi(A\tau)$

(3) $\xi_h = O(q^{-\tilde{c}/24})$ $(\mathrm{Im}\,\tau \to \infty)$
$\tilde{c} = c - \min H$ effective central charge

(4) $\xi_h \, q^{-(h - \frac{c}{24})}$ periodic with period 1

(5) Fourier coeff. of ξ_h are in $\mathbb{Q}$