So $\mathfrak{M}_k(m) =$ Span of all Hecke eigen forms f on $\Gamma_0(m)$, such that their eigenvalues with respect to all Atkin-Lehner-involution is an intrinsic property of f, not just an accident ~~occurring~~ when changing levels.

One of the most striking features of $\mathfrak{M}_k(m)$ is, that it has in some sense the most simplest dimension formula among ~~big enough~~ all subspace of $M_k(m)$ ~~reflecting~~ which are still big enough to reflect all ~~properties~~ various kinds of ~~species~~ occurring ~~on~~ $\Gamma_0(m)$ in weight k. Actually

$$\dim \mathfrak{M}_k(m) = d(m(k-1)) + \frac{1}{2}a$$

$\Big($ $a =$ largest integer s.t. $a^2 | m$, $d(x) = \frac{x}{12} - \frac{1}{3}\chi_3(x) - \frac{1}{4}\chi_4(x)$ $\Big)$.

~~Also the trace formula~~ The same remark applies more general to the trace of Hecke-operators on $\mathfrak{M}_k(m)$.

The only lack of beauty in the above Theorem is the "$-$"-sign, e.g. there are so far no Jacobi forms ~~attached to~~ corresponding to $M_k(1)$ for $k \equiv$ [illegible] mod 4.

Now here a brand-new type of Jacobi forms saves us.

<u>ad (3)</u>:

The most simplest Jacobi forms are, as pointed out before,

$$\vartheta_{m,\rho}(\tau, z) = \sum_{\substack{r \in \mathbb{Z} \\ r \equiv \rho (2m)}} q^{r^2/4m} \zeta^{r} \ldots$$

It is in some ~~sense~~ the prototype. It is especially holomorphic in τ and z. But, moreover it satisfies

$$(8\pi i m\, \partial_\tau - \partial_z^2)\, \vartheta_{m,\rho} = 0.$$

Now to find natural examples of automorphic forms on $J(\mathbb{R})$ satisfying the heat equation, it lies at hand to look at theta series attached to quadratic forms which are not necessarily ~~pl~~definite.