Note that the Theorem implies a lot of ~~open~~ problems:

~~1) Does there exist a Siegel Hecke-eigenform with $\varphi_1 = 0$.~~

~~2) If F is a Siegel Hecke~~

Let F be a Siegel Hecke-eigenform, not "Maass-Spezial".

1) ~~…~~ Can it happen that $\varphi_1 = 0$

2) What is $D_{F,F}(s)$? (Note: $D_{F,F}(s)$ behaves analytically exactly like the Andrianov-Zeta-funct. $Z_F(s)$, but has a pole whereas $Z_F(s)$ has none!)

ad ②

That, what I consider as the Main Theorem of the Theory of Jacobi forms is the following

Theorem (Zagier/~~…~~ 1986)

~~There exist~~

For each ~~…~~ $m \in \mathbb{Z}, m > 0$ the space $J_{k,m}$ is Hecke-equivariantly isomorphic to a certain subspace $\mathfrak{M}^-_{2k-2}(m)$ of $M_{2k-2}(\Gamma_0(m))$.

Here $\mathfrak{M}_{2k-2}(m) \supseteq$ all newforms on $\Gamma_0(m)$ + a nice choice of old forms)

more precisely:

~~Let f be a Hecke-eigenform on $\Gamma_0(m)$, ~~…~~ let g be its pr~~

~~$L(f,s) = \prod_p Q_p($~~

~~Let $g \in M^{new}_{2k-2}(m')$ for some $m' | m$ its progenitor.~~

~~Assume that~~

~~$$\frac{L(f,s)}{L(g,s)} = \prod Q_p($$~~

$$\mathfrak{M}_k(m) = \mathrm{Span}_{\mathbb{C}} \Big\{ f \in M_k(\Gamma_0(m)) \;\Big|\; \begin{array}{l} \exists\, m'|m,\ g \in M^{new}_{2k-2}(m') \text{ such that} \\ L(f,s) = \Big(\prod_{p | \frac{m}{m'}} Q_p(p^{-s})\Big) L(g,s) \text{ with} \\ Q_p(s) = \text{polyn. in } p^{-s} \text{ satisfying} \\ Q_p(s) = p^{t(\frac{k}{2}-s)} Q_p(k-s) \text{ for all } p^t \| \frac{m}{m'} \end{array} \Big\}.$$

$$\mathfrak{M}_k^{\pm}(m) = \{ f \in \mathfrak{M}_k(m) \mid f|W_m := f\left(\tfrac{-1}{m\tau}\right)(\sqrt{m}\,\tau)^{-k} = \pm(-1)^{k/2} f \}$$
$$= \mathrm{Span}\{ f \in \mathfrak{M}_k(m) \mid f \text{ Hecke-eigenform } \& \ L^*(f,s) = (2\pi)^{-s} m^{s/2}\Gamma(s) L(f,s) = \pm L^*(f,k-s)\}$$