A more thorough study of this situation gives the basic

__Prop__ The correspondence $\phi \longrightarrow \sum_{\mu=1}^{2m} h_\mu \otimes \vartheta_{m,\mu}$ defines an isomorphism

$$J_{k,m} \xrightarrow{\cong} \Big(M_{k-1/2}(\Gamma(4m)) \otimes \mathrm{Span}\{\vartheta_{m,\mu}\ (\mu=1,\dots,2m)\}\Big)^{SL_2(\mathbb{Z})}$$

This is the ~~basic~~ main key to the ~~connection~~ understanding of the connection of Jacobi forms with modular forms of 1/2-integral weight. However, I do not want to go into this here. But it may ~~help~~ give you ~~to ob~~ an intuitive understanding of basic facts about Jacobi forms:

- dim $J_{k,m} < \infty$, actually $\frac{mk}{6} + O(1)$ $(k \to \infty)$ ~~$d(m(k-1)) + \frac{1}{2}d$ for $k \geq 2$, 0 for $k<2$~~
  (Codim $\approx \frac{x}{12} - \frac{1}{3}\chi_3(x) - \frac{1}{4}[\chi_4(x)]$, $x =$ [illegible] $/m$)
- there exists the notion of Eisenstein series, cusp forms
- there exists a Hecke theory for ~~Jacobi~~ forms
- there exist a "Petersson scalar product" $\langle \phi | \psi \rangle$ of $\phi, \psi \in J_{k,m}^{cusp}$.

~~Back to Siegel modular forms: Any Siegel modular form can be~~

Finally an example:

$\bigoplus_{k \in \mathbb{Z}} J_{k,1}$ is a free module over $\bigoplus_{k \in \mathbb{Z}} M_k(SL_2(\mathbb{Z}))$ ~~in two gen~~ of rank 2. Generators are

$$E_k^J = \sum_{\gamma \in SL_2(\mathbb{Z})^J_\infty \backslash SL_2(\mathbb{Z})^J} 1|_{k,1}\gamma = * \sum_{\substack{r,\Delta \in \mathbb{Z},\ \Delta<0 \\ r^2 \equiv \Delta\ (4)}} \left(\sum \frac{\#\{x \bmod 2n \mid x^2 \equiv \Delta\ (4n)\}}{n^s} \Big|_{s=k-1} \right) |\Delta|^{k-3/2}\, q^{\frac{r^2-\Delta}{4}} \zeta^r$$

$$= \cancel{4}\ \eta(\tau)^{-6} \Big\{ E_k(\tau) \sum_{\substack{s,r \in \mathbb{Z} \\ s \not\equiv r\ (2)}} s^2 (-1)^r q^{(s^2+r^2)/4} \zeta^r$$

$$- \frac{1}{\pi i k} E_k'(\tau) \sum (-1)^r q^{(s^2+r^2)/4} \zeta^r \Big\} \in J_{k,1} \quad (k=4,6)$$

where $\eta(\tau) = q^{\frac{1}{24}} \prod_{n \geq 1} (1-q^n)$, $E_k = 1 - \frac{2k}{B_k} \sum \sigma_{k-1}(n) q^n$.