~~Hence~~ the following I shall try to explain the highlights of the theory of Jacobi forms so far obtained. I hope that then the above diagrams will become self-explanatory. However, I shall not speak about the listed applications of the theory of Jacobi forms. Also I shall not speak about results about Jacobi forms of higher degree (genus), e.g. by Murase, , Ziegler (88)

~~For the main talk I have chosen~~

I have partitioned ~~the~~ the main talk into four parts

1. Jac forms and Siegel mod. forms
2. Jac forms and ellipt. mod forms of int. weight
3. skew-hol. Jac forms
4. \# 2 continued.

ad ①

Let $F(\tau,z,\tau') \in M_k(Sp_2(\mathbb{Z}))$, $(\tau,\tau' \in \mathfrak{H} = \{\tau \mid \mathrm{Im}\,\tau > 0\},\ z \in \mathbb{C},\ \mathrm{Im}\,\tau\,\mathrm{Im}\,\tau' - (\mathrm{Im}\,z)^2 > 0)$. ~~Here the notation is as follows~~

Then F is especially periodic with respect to τ' and thus we can expand F into a Fourier series with resp. to τ'. This is the so called Fourier – Jacobi – expansion: $F(\tau,z,\tau') = \sum_{n \geq 0} \varphi_n(\tau,z)\, e^{2\pi i n \tau'}$.

It is clear that the automorphic behaviour of F with respect to $Sp_2(\mathbb{Z})$ implies also some automorphic behaviour of the φ_n with respect to whatever. A function behaving exactly like φ_n, i.e. a function which could possibly occur as a φ_n for some F, is called a Jacobi form. Now, a more thorough study of this situation gives the following.

Define

$$J(\mathbb{R}) = \text{Jacobi group} = SL_2(\mathbb{R}) \ltimes \mathbb{R}^2 \cdot S^1;$$

identify $SL_2(\mathbb{R})$, $\mathbb{R}^2$, S^1 with its canonical images in $J(\mathbb{R})$, so that any $\eta \in J(\mathbb{R})$ can uniquely be written as

$$\eta = A[\lambda,\mu]s \quad (A \in SL_2\mathbb{R},\ (\lambda,\mu) \in \mathbb{R}^2,\ s \in S^1).$$

The multiplication law is

$$\eta\eta' = AA'[(\lambda,\mu)A' + (\lambda',\mu')]\, ss'\, e\left(\begin{vmatrix} (\lambda,\mu)A' \\ (\lambda',\mu') \end{vmatrix}\right) \qquad (e(\ldots) = e^{2\pi i(\ldots)}).$$