($E_{\ell-1} = 1 - \frac{2(\ell-1)}{B_{\ell-1}} \sum_n \sigma_{\ell}(n) q^n$ (3)
von Staudt: $\ell B_{\ell-1} \equiv -1 \bmod \ell$) $E_{\ell-1} \underset{\ell}{\equiv} 1$ "von-Staudt" congruence)

Thm (Serre – Hida – Katz – ... ?)
Any mod. form of level ℓ^n with integral coeff. is congruent mod ℓ to a mod form of level 1.

In the rest of the talk we indicate how to prove a more concrete version of the latter theorem in the case of theta series.
The proof also admits several generalisations (level different from 1, prime power congruences).

Thm Let
$Q(x)$ pos. def. integral quad. form of rank $r = r(Q)$
$Q(x) = \frac{1}{2} x^t F x$, F symmetric.
$e = e(Q) = (\text{sum of elementary divisors of } F)/2$
Assume $s = s(Q) = \ell^n$ (~~for some prime~~ $n > 0$)
($s =$ smallest pos. int. s.t. sF^{-1} even integral).
Then there is a $f \in M_{e(Q)}$ such that

$$\Theta_Q = \sum_{x \in \mathbb{Z}^r} q^{Q(x)} \underset{\ell}{\equiv} f.$$

use $(-1)^{r/2} d(Q) \underset{4}{\equiv} 1$

Rem. e even integral, $e \equiv \begin{cases} \frac{r(Q)}{2} & d(Q) = \square \\ \frac{r(Q)}{2} + \ell - 1 & d(Q) \neq \square \end{cases}$
Ex $\ell \underset{4}{\equiv} -1$, $Q = [1, 1, \frac{\ell+1}{4}]$ has $e(Q) = \frac{1+\ell}{2}$

$\Theta_{[1,1,\frac{\ell+1}{4}]} \bmod \ell \in M_{\frac{\ell+1}{2}}(\mathbb{Z}) \bmod \ell$.