<u>Exercise</u> Show for $k=2$ and $\gcd(l,N)=1$, $v \in \Gamma_0(N)$:

$$\lambda_{T(l)f}(v) = \sum_{\substack{ad=n \\ d>0 \\ b \bmod d}} \sum_{\mu=0}^{r(b/d)} \lambda_f\left(v \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}^{-1} C_{\frac{b}{d},\mu} \right)$$

where $\frac{b}{d} = [a_0, \dots, a_r]$, $r(b/d) = r$

$$C_{\frac{b}{d},\mu} = \begin{pmatrix} a_0 & 1 \\ 1 & 0 \end{pmatrix} \cdots \begin{pmatrix} a_r & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & (-1)^{\mu-1} \end{pmatrix}.$$

$$\cancel{\Gamma_{l^2} f \sum_{M \in \Gamma \backslash M(l)} f|M = l^2 \sum_{M} \int_{MA_0}^{MA_\infty} f = \sum_{N} \int_{AN_0}^{AN_\infty} f}$$

~~(since $MA = C_M A N_M$ for suitable $C_M \in \Gamma$, $N_M \in M(l)$ etc...)~~

$$\cancel{= l^2 \sum_{N \text{ diagonal}} \int_{AN_0}^{AN_\infty} f = l^2 \sum_{\substack{ad=l \\ b \bmod d}} \int_{b/d}^{\infty} f}$$

$$\cancel{\infty - \frac{b}{d} =}$$

Define $T(l)\lambda$ by the right hand side "with f dropped". For $\lambda \in X_2(\Gamma_0(N))$.

Then

<u>Thm.</u> $X_0(\Gamma_0(N))^+$ resp. $X_0(\Gamma_0(N))^-$ are Hecke equivariantly isomorphic to subspaces $M^\pm_2(\Gamma_0(N))$, respectively. M^+ and M^- contain $S_2(\Gamma_0(N))$ and $M^+ + M^- = M_2(\Gamma_0(N))$.