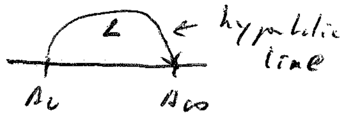
An unnecessary explicit map

For $f \in S_2(\Gamma)$, $A \in SL(2,\mathbb{Z})$ set

$$\lambda_f : G_\Gamma \longrightarrow \mathbb{C}, \quad \lambda_f(\Gamma A) = \int_{A0}^{A\infty} f(z)\,dz$$

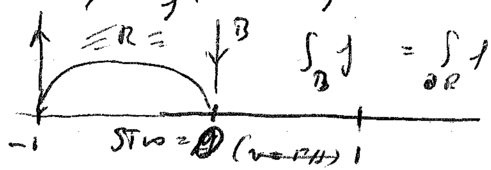


$$= \int_L f(z)\,dz$$

[Note: $\lambda(CA) = \int_{CA0}^{CA\infty} f(z)\,dz = \int_{A0}^{A\infty} f(z)\,dz \circ A = \lambda(A)$ for $C \in \Gamma$].

Note

$$\lambda_f(v) + \lambda_f(vS) = 0, \quad \lambda_f(v) + \lambda_f(vR) + \lambda_f(vR^2) = 0$$



$$\int_B f = \int_{\partial R} f = \int_R df = 0$$

Thm. $f \to \lambda_f^+$ resp. $f \mapsto \lambda_f^-$ define injections $S_2(\Gamma) \hookrightarrow X_2^+(\Gamma)$, $X_2(\Gamma)^-$

Similarly for $k \geq 2$, $f \in S_k(\Gamma)$, $A \in SL(2,\mathbb{Z})$

$$\lambda_f(A) := \int_{A0}^{A\infty} f(z)(X - zY)^{k-2}\,dz$$

Exercise: $\lambda_f(GA) = G \cdot \lambda_f(A)$ for $G \in \Gamma$.

Hint: $(Aw - Az)(cw+d)(cz+d) = w - z$.