admissible labeling of degree n: $\lambda: SL(2,\mathbb{Z}) \longrightarrow \mathbb{Z}[X,Y]_n$
$$\lambda(GX) = G\cdot\lambda(X)$$
s.t. $\lambda(X)+\lambda(XS)=0$
$$\lambda(X)+\lambda(XR)+\lambda(XR^2)=0.$$

$X_k(\Gamma) = \mathbb{Z}$-module of admissible labelings of degree $k-2$.

<u>Note</u> $X_2(\Gamma)$ newly defined can be identified with "old $X_2(\Gamma)$" in obvious manner.

<u>Example</u> $\Gamma_{SL(2,\mathbb{Z})}$:

$X_2(SL(2,\mathbb{Z}))$: $\lambda : {}_{SL(2,\mathbb{Z})}\backslash SL(2,\mathbb{Z}) \to \mathbb{Z}$ (i.e. λ const.)
$$(1+1)\lambda = 2\lambda = 0 \quad \text{, i.e. } \lambda = 0.$$

$X_k(SL(2,\mathbb{Z}))$: $\lambda : SL(2,\mathbb{Z}) \to \mathbb{Z}[X,Y]_{k-2}$
$$\lambda(X) = X\cdot\lambda(1)$$
$$\lambda(1) + S\lambda(1) = 0$$
$$\lambda(1) + R\,\lambda(1) + R^2\lambda(1) = 0$$
i.e. $\lambda(1) \Longrightarrow f : \{f \in \mathbb{Z}[X,Y]_{k-2} : f(X,Y) + f(-Y,X) = 0$
$$f(X,Y) + f(Y,Y-X) + f(-X+Y,-X) = 0\}$$
$$\cong X_k(SL(2,\mathbb{Z})).$$

<u>Exercises:</u> 1) Determine the first $X_k(SL(2,\mathbb{Z}))$

2) Prove $\dim X_k(SL(2,\mathbb{Z})) = \dim S_k(SL(2,\mathbb{Z}))$.

Submit before next Thursday.