$\Gamma = \Gamma_0(p)$, p prime

f. pts of S on $C_p = \mathbb{P}^1(\mathbb{F}_p)$: $\frac{-1}{a} = a$, i.e. $a^2 = -1$

$\# = 1 + \left(\frac{-4}{p}\right)$

f. pts of R on $C_p = \mathbb{P}^1(\mathbb{F}_p)$: $\frac{1}{1-a} = a$, i.e. $a^2 - a + 1 = 0$

$\# = 1 + \left(\frac{-3}{p}\right)$

$$d_{\Gamma_0(p)} = \frac{p+1}{6} - \frac{1}{2}\left(1 + \left(\frac{-4}{p}\right)\right) - \frac{2}{3}\left(1 + \left(\frac{-3}{p}\right)\right) + 1$$

$$= 1 + 2 \text{ genus of } X_0(p)$$

<u>Ex</u> For arbitrary Γ (with $-1 \in \Gamma$):

$$\boxed{d_\Gamma = \dim M_2^{Eis}(\Gamma) + 2 \dim S_2(\Gamma).}$$

[Complete the above calculation and compare the result with Shimura's book formula for $\dim M_2(\Gamma)$, ...]

<u>Labels of higher weight</u>

$SL(2,\mathbb{Z})$ acts on $\mathbb{C}[X,Y]_n$ via $(Af)(X,Y) = f((X,Y)A^{-1})$

admissible labels: pairs (A, f) such that

- $A \in V$ (i.e. A represents the coset $v \in \Gamma \backslash SL(2,\mathbb{Z})$)
- $f \in \mathbb{C}[X,Y]_n$
- v_1, v_2 joined blue, say $v_1 = (A, f)$, $v_2 = (GAS, g)$, $G \in \Gamma$, then $f + Gg = 0$
- $v_1\, v_2\, v_3$ joined red, say $= (A,f), (GAR, g), (HAR^2, h)$ $(G, H \in \Gamma)$ then $f + G\cdot g + H\cdot h = 0$.