同濟大學 教师备课用纸

Dimension calculation

$SL(2,\mathbb{Z})$ acts on $\mathbb{C}[G_r] \circlearrowleft$, $X_2(r)_{\mathbb{C}} := \mathbb{C}[G_r]^*$

$$X_2(r)_{\mathbb{C}} = \mathrm{Ker}(1+S) \cap \mathrm{Ker}(1+R+R^2) \subseteq X$$

$$X_2(r)_{\mathbb{C}}^* = \mathrm{Ker}(1+S)^* + \mathrm{Ker}(1+R+R^2)^* \subseteq X^* = \mathbb{C}[G_r]$$
$$= \mathrm{Ker}(1-S) + \mathrm{Ker}(1-R) \subseteq X^*$$

[Note: $f \in \mathrm{End}(V)$, $\mathrm{ord} f = n$

$$\mathrm{Ker}(g)^* = \{y \in V^* \mid (x,y) = 0 \ \forall x \in \mathrm{Ker}\, g\} \quad ((x,y) := y(x))$$

$(1+f+\dots+f^{n-1} =: g)$ $\quad = \mathrm{Im}\, g^* = \mathrm{Ker}(1-f)$.]

$$d_r = \dim X^* - (\dim \underbrace{\mathrm{Ker}(1-S) + \dim \mathrm{Ker}(1-R)}_{\subseteq X^*}) + \dim \mathrm{Ker}(1-S) \cap \mathrm{Ker}(1-R)$$

$$= \dim X^* - \dim X^{*S} - \dim X^{*R} + \dim X^{*SL(2,\mathbb{Z})}$$

$$= \# G_r - \dim \mathbb{C}[G_r]^S - \dim \mathbb{C}[G_r]^R + 1$$

[Note: $\mathbb{C}[G_r]^{SL(2,\mathbb{Z})} = \sum_{v \in G_r} (v)$ since $SL(2,\mathbb{Z})$ acts transitively on G_r]

$$\mathbb{C}[G_r]^S = \mathrm{span}_{\mathbb{C}} < \sum_{v \in \sigma} (v) \mid \sigma \in G_r/<S> >$$

$$\dim \mathbb{C}[G_r]^S = \frac{1}{2} \# G_r + \frac{1}{2} \# \underset{\text{on } G_r}{\text{fixed pts of } S}$$

Similarly

$$\dim \mathbb{C}[G_r]^R = \frac{1}{3} G_r + \frac{2}{3} \# \underset{\text{on } G_r}{\text{fixed pts. of } R}$$

$$d_r = \frac{1}{6} \# G_r - \frac{1}{2} \underset{\text{on } G_r}{\text{f.pts of } S} - \frac{2}{3} \underset{\text{on } G_r}{\text{fixed pts of } R} + 1$$