



Labels

$X_{2n}(\mathbb{Z})$ labels on C_n such that:

sum of labels at vert. joined by same color = 0

Labeling: map $\lambda: C_n \rightarrow \mathbb{Z}$

$$X_2(P) := \left\{ \lambda: C_n \rightarrow \mathbb{Z} : \begin{array}{l} \lambda(v) + \lambda(vS) = 0 \\ \lambda(v) + \lambda(vR) + \lambda(vR^2) = 0 \end{array} \right\}$$

for all $v \in C_n$

Note: $X_2(P)$ free \mathbb{Z} -module ($\cong \mathbb{Z}^{\#C_n}$)

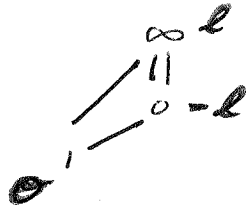
• Set $X_2(P)_R = \{ \lambda: C_n \rightarrow R : \dots \}$

then $X_2(P)_R = R \otimes X_2(P)$.

$$d_P := \text{rank } X_2(P) = \dim_{\mathbb{Q}} X_2(P)_{\mathbb{Q}} = \dim_{\mathbb{C}} X_2(P)_{\mathbb{C}}$$

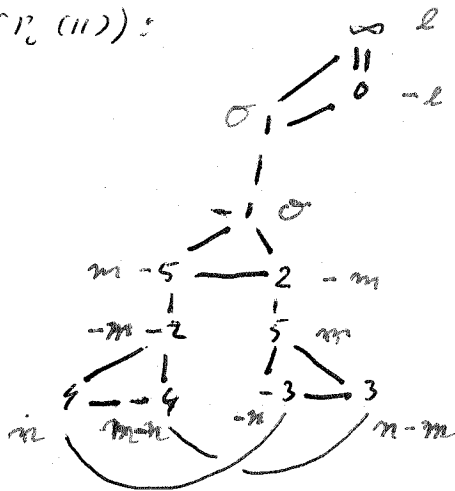
Examples:

$X_2(P_2(2)):$



$$d_{P_2(2)} = 1$$

$X_2(P_2(11)):$



$$d_{P_2(11)} = 3$$