Siegen – Oberseminar – 17+24–07–2008 ①

同濟大學 教师备课用纸

Computation of modular forms via periods

Graphs

$\Gamma \leq SL(2,\mathbb{Z})$, assume $\boxed{-1 \in \Gamma}$

Graph G_Γ:

Vertices: $\Gamma \backslash SL(2,\mathbb{Z})$

Blue edges: $(\Gamma A, \Gamma B)$ iff $\Gamma A S = \Gamma B$

Red edges: $(\Gamma A, \Gamma B)$ iff $\Gamma A R = \Gamma B$ or $\Gamma A R^2 = \Gamma B$

$[\ S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},\ T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},\ R = ST = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix},\ R^2 = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$

$S^2 = -id,\ R^3 = -id\ ]$

Examples

$\Gamma_0(N) \backslash SL(2,\mathbb{Z}) \xrightarrow{\sim} \mathbb{P}^1(\mathbb{Z}/N\mathbb{Z}) = \{\underbrace{(x,y) \in \mathbb{Z}/N\mathbb{Z} \mid \gcd(x,y,N) = 1}_{(\mathbb{Z}/N\mathbb{Z})^*}\}$

$\underset{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}{\Gamma_0(N) A} \longmapsto [0:1]A = [c:d]$

$\Gamma_0(N) A \longleftrightarrow [c:d]$, $\Gamma_0(N) A X \longleftrightarrow [c:d]X$

$[x:y]\,S = [y:-x]$

$[x:y]\,R = [y:-x+y]$, $[x:y]\,R^2 = [-x+y,-x]$

$N = p$ = prime

$\mathbb{F}_p \hookrightarrow \mathbb{P}^1(\mathbb{Z}/p\mathbb{Z})$, $\infty := [1:0]$, $\mathbb{P}^1(\mathbb{F}_p) = \mathbb{F}_p \cup \infty$

$a \longmapsto [a:1]$

$aS = -1/a$ $\left(\cancel{-1/0 = \infty}\right)$ $0S = \infty$, $\infty S = 0$ $\begin{pmatrix} \text{conv.} \\ \pm 1/0 = \infty \\ \pm 1/\infty = 0 \end{pmatrix}$

$aR = \frac{1}{1-a}$, $aR^2 = 1 - 1/a$ $\quad \infty \xrightarrow{R} 0 \xrightarrow{R} 1 \xrightarrow{R} \infty$