Now, for a discriminant $D < 0$ (3)

$$f = \frac{h(D)}{w(D)} + \sum_{n=1}^{\infty} \sum_{d|n} \left(\frac{D}{d}\right) q^n \in M_1(\Gamma_0(D), (\tfrac{D}{\cdot}))$$

But

$$f = \text{const} + \sum_{x,y>0} \left(\frac{D}{x}\right) q^{xy} \qquad \text{weight } 1$$

theta series of $Q(x,y) = xy$

In the Part II of computational section, we saw

$$g = \text{const} + \sum_{\substack{N|a,\, 2N|b \\ b^2-4ac=D}} \chi\left(\frac{a}{N}\right)\left(\frac{-4Nt}{a/N}\right)(\operatorname{sign} Q(\infty) - \operatorname{sign} Q(0))\, q^{D} \in M_{3/2}(\Gamma_0(4N), \chi)$$

But

$$g = \text{const} + \sum_{\substack{x,y,z \in \mathbb{Z} \\ xz<0}} \alpha(x) \left(\frac{-4Nt}{|x|}\right) \cancel{\operatorname{sign}(x)}\, q^{Ny^2 - xz} \qquad \text{weight } 3/2$$

theta series of $Q(x,y,z) = Ny^2 - xz$.

Finally in Part I, we saw, that for a labelling λ of $G_{\Gamma_0(N)}$ we have

$$h = \text{const} + \sum_{l=1}^{\infty} \sum_{\substack{M = \left(\begin{smallmatrix} a & b \\ c & d\end{smallmatrix}\right) \\ \det M = l \\ a > b \geq 0 \\ d > c \geq 0}} \lambda([u:v]M)\, q^l \quad ([u:v] \in \mathbb{P}^1(\mathbb{Z}/N\mathbb{Z})) \in M_2(N)$$

But

$$h = \text{const} + \sum_{\substack{x,y,z,w \in \mathbb{Z} \\ x > z \geq 0 \\ y > w \geq 0}} \lambda(xu + wv, zu + yv)\, q^{xy - zw} \qquad \left(M \sim \begin{pmatrix} x & z \\ w & y\end{pmatrix}\right)$$

weight 2

theta series of $Q = xy - zw$.

<u>Questions</u>: Given an indefinite form Q, for what domains (cones?) $\mathcal{D}$ is

$$\sum_{x_1, \dots, x_t \in \mathbb{Z} \cap \mathcal{D}} q^{Q(x_1, \dots, x_t)}\, \psi(x_1, \dots, x_n)$$

a modular form, where ψ is a function depending on x_i mod N for some N.