2

If $g_1, \dots, g_t$ is a system of generators of $M_*(\Gamma)$, then $p \longmapsto p(g_1, \dots, g_t)$ defines an isomorphism

$$\mathbb{C}[x_1, \dots, x_t]/_{I} \xrightarrow{\cong} M_*(\Gamma)$$

for a suitable ideal I.

Let $g_1, \dots, g_t$ a <u>minimal</u> set of generators. The g_i are not unique, but

$$w_k = \# \text{ of } g_i \text{ of wt } k \qquad (k = 1, 2, 3, \dots)$$

is <u>unique</u>.

Look at $\Gamma_0(N)$ for $N = 1, 2, \dots$.

— table —

<u>Question</u>. For $\Gamma_0(N)$,
Is it true that we can always find generators in weights 2 and 4 and 6?
More generally, for $\Gamma_0(N)$, what are the numbers $w_2, w_4, w_6, \dots$?

<u>Problem 2</u>

If $Q \in \mathbb{Z}[x_1, \dots, x_r]_2$ is positive definite, then, for any n,
$\#\{(x_1, \dots, x_r) \in \mathbb{Z}^r \mid n = Q(x_1, \dots, x_r)\}$ is finite.
(since "$Q(x_1, \dots, x_r) \leq n$" is compact). and $= O(n^r)$.
Hence

$$\Theta_Q := \sum_{x_1, \dots, x_r \in \mathbb{Z}} q^{Q(x_1, \dots, x_r)} \qquad \text{converges for } \tau \in \mathfrak{H}$$

and, in fact,

$$\Theta_Q \in M_{r/2}\left(\Gamma_0(\text{level of } Q), \left(\frac{\text{disc } Q}{\cdot}\right)\right).$$