§5 Interpretation of the formula for $L_{f,X}(c)$

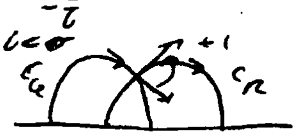
$\mathcal{L}$ = set of oriented hyperbolic lines

$= \{ c_Q \mid Q \in \mathbb{R}[X]_{\deg \le 2},\ \mathrm{disc}\,Q > 0 \}$

c_Q : $\hat{Q}(z) = 0$, oriented $\lambda_Q^+ \curvearrowright \lambda_Q^-$: $\lambda_Q^{\pm} = \dfrac{-b \pm \sqrt{\mathrm{disc}}}{2a}$

$a \neq 0$; $a = 0$: $-\frac{c}{b}$, $b > 0$, $b < 0$

define intersection #:



$\bullet : \mathbb{Z}[\mathcal{L}] \otimes \mathbb{Z}[\mathcal{L}] \longrightarrow \mathbb{Z}$

$$c_Q \cdot c_R := \tfrac{1}{2}\left(\mathrm{sign}\, Q(\lambda_R^+) - \mathrm{sign}\, Q(\lambda_R^-)\right)$$

antisymmetric, $GL(2,\mathbb{R})^+$-invariant, factors through a pairing of

$$\mathbb{Z}[\mathcal{L}]/_{\ker\partial} \otimes \mathbb{Z}[\mathcal{L}]/_{\ker\partial} \longrightarrow \mathbb{Z}$$

where

$$\mathbb{Z}[\mathcal{L}] \xrightarrow{\partial} \mathbb{Z}[\mathbb{P}^1\mathbb{R}]^0 \longrightarrow 0$$
$$c_Q \longmapsto (\lambda_Q^-) - (\lambda_Q^+)$$

Let $c = \sum c_s (s) \in \mathbb{Z}[\mathbb{P}^1\mathbb{R}]^0$, $\tilde{c} \overset{\partial}{\mapsto} c$

Then

$$\sum_s c_s\ \mathrm{sign}(Q(s)) = c_Q \cdot \tilde{c}$$

θ-th. FC of $L_{f,X}(c)$

= inters. # of proj. of $\sum_{Q \in F/\Gamma_0(2N)} \chi_f(Q)\, c_Q$, $\tilde{c}$

onto $X_0(2N)$.