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Hence

$$\boxed{L_{t,X}(C) \underset{M^{\text{hol}}_{3/2}(4N,\,X e_{4N})}{=} W_{4N}\circ \ell\left(\int_{C^+}\Theta_{t,X}(\cdot,z)\,d\bar z\right).}$$

(Actually $\in$ ( )$^{\text{hol. proj.}}$ of right hand side, but turns out to be already hol.)

§4 Computation of $L_{t,X}$

We use

Lemma $\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0$ is a free $\mathbb{Z}[SL(2,\mathbb{Z})]$-module of rank 1. More precisely one has

$$\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0 = \mathbb{Z}[SL(2,\mathbb{Z})]\cdot\big((\infty)-(0)\big)$$

proof "Manin trick".

Thus it suffices to compute, for $A\in SL(2,\mathbb{Z})$:

$$F_{t,x}(A) = \cancel{-\int_{A0}^{A\infty}} + L_{t,X}\left(\text{co-inv class of } (A\infty - A0)\otimes 1\right)$$

$$= +\left(\int_{\infty}^{A\infty} - \int_{\infty}^{A0} + \int_{\infty}^{-A\infty} - \int_{\infty}^{-A0}\right)\left[W_{4N}\,\ell\,\Theta_{t,X}(\cdot,z)\right]dz$$

$$= +\left(\int_{A0}^{A\infty} + \int_{-A0}^{-A\infty}\right)\underbrace{\left[W_{4N}\,\ell\,\Theta_{t,X}(\cdot,z)\right]dz}_{\sigma}$$

$$\left(= \int_0^\infty \left[W_{4N}\,\ell\,\Theta_{t,X}(\cdot,Az)\right]dAz + \left[W_{4N}\,\ell\,\Theta_{t,X}(\cdot,A^*z)\right]dA^*z\right) = \int_0^\infty \sigma\circ A + \sigma\circ A^*$$

where $A^* = \begin{pmatrix} a & -b\\ -c & d\end{pmatrix}$ if $A = \begin{pmatrix} a & b\\ c & d\end{pmatrix}$